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ABSTRACT

This document is intended as a teacher's guide to the three course options open to 10th-grade students in Manitoba. The material opens with an overview, which, after an introduction, looks at the structure for senior high school mathematics, a description of senior high courses, the use of mathematics, the general goal, objectives for 7th through 9th grade mathematics, and objectives for the 10th grade options. The next section ("Implementation") discusses a rationale; general goals for 04, 01, and 00 mathematics; the guide's sequencing and time allotments; the role of textbooks; recommended uses of calculators and computers; evaluation; special needs students; and implementation. The bulk of the document looks at the three 10th grade courses. Each section detailing individual grade 10 offerings discusses general goals, the type of student taking the course, course features, structure, and core and optional topics. The objectives and notes to teachers are presented in detail. (MP)

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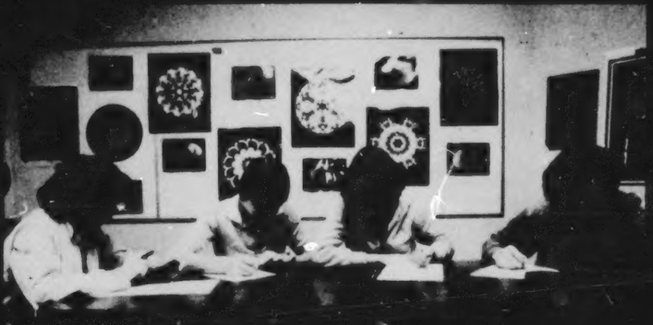

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The Mathematics 104, 101, 100 guide replaces the Mathematics 101 (1972), Business Mathematics 102 (1971) and Temporary Revision (1978), Industrial Mathematics 103 (1971), Algebra 100 (1972), Geometry 100 (1974), and the revised interim edition issued in 1981. It has been developed as part of an overall revision of the mathematics program for Manitoba schools and is designed to articulate with existing K-9 guides. The final form is based on the interim document and incorporates the suggestions and input of teachers. Mathematics 104, 101, 100 becomes the approved provincial guide for mathematics in grade 10, effective September, 1982.

There are five main sections in this guide: 1) a general overview of the mathematics program, 2) a description of program implementation, 3) the Math 104 outline, 4) the Math 101 outline, 5) the Math 100 outline.

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overview

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OVERVIEW

INTRODUCTION

The Mathematics Working Party (K-12) [MWP(K-12)] was established in conjunction with the Articulation Council, and the Elementary, Middle Years and Senior High Program Review Committees in March 1977 by the Department of Education. Its terms of reference were defined this way: "to review the goals, content, methodology, and resources of all currently approved and developing provincial programs in mathematics and to establish the necessary committees/curriculum structures to complete the revision of the K-12 programs in mathematics within an articulated K-12 framework."

The K-6 Mathematics guide (1978) encourages the teacher to use an activity-oriented approach to promote equally the acquisition of basic skills and the understanding of concepts. To develop these skills and concepts, materials at the beginning are concrete and manipulative; for later reinforcement, materials are concrete and semi-concrete. These two stages enable the learner to proceed gradually to the abstract or symbolic stage.

The 7-9 Mathematics guide (1979) recommends that students develop both a prescribed set of basic skills and an understanding of the structure and vocabulary of mathematics. To promote the development of these skills and concepts--essential to students in subsequent grades--the guide suggests that terminology be de-emphasized, understanding stressed, and appropriate practice recommended. While material outlined in the guide for grades 7 and 8 is expected to be taken in its entirety, recommended options make up 30% of the grade 9 course (three out of five options, selected according to student interest and ability). But consistent throughout the 7 to 9 program, to be stressed at every opportunity, are four major areas entitled "unifying themes": arithmetic operations, estimation, problem solving and consumer mathematics, and measurement. The program is further unified since work at each grade level is designed to include a maintenance program.

Meetings of senior high mathematics teachers were held in Winnipeg, Thompson, The Pas, Dauphin, Brandon, Morris, and Teulon in the spring of 1978 to obtain a sample of opinion on the following topics: the structure of high school mathematics programs, of geometry, consumer mathematics, business-vocational mathematics, and the mathematical needs of non-college-bound students. Responses to questionnaires distributed to teachers, superintendents, and school trustees indicated agreement on the following points:

- Some form of streaming is necessary in high school mathematics to provide for the varying levels of ability, needs, and interests in students. Response to this issue from the Manitoba Association of School Superintendents (MASS) states that "subject matter appropriate for different students will vary. It is not just the speed at which material can be assimilated by different pupils, but the actual subject matter itself which is required for differing needs."
- The 01 program requires immediate modification. Instead of paralleling the existing 00 program, it should meet the needs of students not intending to pursue post-secondary study in mathematics by applying more closely to practical problems that arise in everyday life.
- The 00 program should remain substantially as it is. The amount and rigour of formal geometry should be reduced, although geometry is still to be retained as an important part of a revised course.
- Consumer mathematics should be studied by all students.

STRUCTURE FOR SENIOR HIGH SCHOOL MATHEMATICS

The Proposal for High School Mathematics Revision, prepared by the Mathematics Working Party (K-12) and presented to the Curriculum Policy Review Council in January 1979, identified three levels of senior high mathematics (04, 01, and 00), each containing consumer mathematics. This proposal, which reflected data obtained in the previously mentioned survey, in interface meetings with post-secondary education representatives, and from the MWP(K-12) membership, was approved and forms the basis of the current 10-12 Mathematics revision. The following chart outlines the course offerings for senior high school mathematics.

	GRADE 10	GRADE 11	GRADE 12
COURSE "04"	Mathematics 104	Mathematics 204	Mathematics 304
COURSE "01"	Mathematics 101	Mathematics 201	Mathematics 301
COURSE "00"	Mathematics 100	Mathematics 200	Mathematics 300
			Math Topics 30X*
			Statistics 30X*

*Presently designated as 305.

DESCRIPTION OF THE SENIOR HIGH COURSES

04 Mathematics - full credit courses (60% core, 40% options in each of grades 10, 11 and 12) designed to concentrate on the application of mathematical skills and concepts to everyday situations. Skills covered include information processing, estimation and the use of calculators, computation, and problem solving in areas encountered by the student at home, at work, and at leisure. These courses are intended as an option for any student with or without work education. However, students intending to pursue post-secondary education should be advised against taking them.

01 Mathematics - full credit courses (50% core, 50% options in each of grades 10 and 11, and 70% core, 30% options in grade 12) designed to include practical applications to current problems from the business, consumer, trade, and vocational areas. The current 01 program amalgamates previous 01 (general), 02 (business), and 03 (vocational) mathematics courses. It is truly a general mathematics program, incorporating the best from several branches of mathematics, and leading to a range of post-secondary education programs, vocational training and employment, as well as to more general life skills. Since it is a general program, not algebra based, competency in algebra is not essential for successs. 01 Mathematics is designed to expand future student options by introducing a variety of mathematics topics, to benefit smaller schools by making business and vocational skills part of the 01 Mathematics package, and to allow larger schools the option of identifying separate classes of 01 Mathematics with options dedicated to business, vocational or general mathematics.

00 Mathematics - full credit courses (85% core, 15% options in each of grades 10 and 11, and 80% core, 20% options in grade 12) designed for students wishing to pursue further studies in either mathematics or science-related programs at post-secondary institutions. Both Mathematics 100 and 200 contain 50% algebra, 40% geometry, and 10% consumer math. Mathematics 300 contains 80% core and 20% option topics selected from algebra, analytic geometry and trigonometry. To make these courses more meaningful and relevant, a greater emphasis has been placed on problem solving and on practical applications of content under consideration.

USE OF MATHEMATICS

It is important to study mathematics because of its many applications to everyday life and to specialized scientific work. A knowledge of mathematics is necessary if students are to have an awareness and an understanding of their environment and to enjoy a fuller life.

With this in mind, the Mathematics Working Party (K-12) suggests that mathematics be considered:

1. A mode of communication, with a language all its own. For students to communicate successfully in mathematics, they must learn the basic concepts without having to meet demands for logical rigour. The language acquired should be used to describe relationships observed in the real world.

At a later stage, symbols should be introduced to record these relationships.

2. A tool. After a mathematical system has been introduced as a language, it should be used as a tool to answer practical questions, thus demonstrating to students that to function in society they need basic skills in arithmetic, algebra, geometry, and statistics. Students should be capable of identifying a problem and employing an organized process leading to its solution. This involves applying fundamental operations and information processing skills--such as collecting, organizing and interpreting data--to contexts both familiar and unfamiliar. Realistic problems relating to the students' environment should be sought to foster consumer awareness, and vocational and business applications should be stressed at every opportunity.

3. A source of aesthetic enjoyment. Mathematics can be considered an aesthetic discipline like music. Many students can develop an appreciation for mathematics through the discovery of patterns in numbers and in created or naturally occurring designs. Students should be encouraged to reproduce these as well as to create their own. Methodologies and techniques conducive to developing positive inquiring attitudes toward mathematics should be employed.

Students who are most successful in developing skills and understanding concepts in mathematics have learned to recognize and appreciate the three ways in which the subject can be used. Classroom emphasis on those uses cannot always be equal, however, but must vary depending on the nature of the topic and the interests and abilities of the students.

GENERAL GOAL OF MATHEMATICS

The aim of mathematics is to foster maximum and continuous growth in the following behavioural areas: the cognitive (factual), the affective (feelings, perceptions), and the psychomotor (movements). This growth will be accomplished when mathematical content is used to develop processes such as the following:

1. The process of communicating: using mathematical language and symbolism
2. The process of recalling mathematical skills
3. The process of comprehending mathematical principles
4. The process of seeing relationships in mathematics
5. The process of generalizing: finding common relationships among mathematical concepts
6. The process of making applications: applying generalizations to new situations
7. The process of learning self-direction: searching for mathematical ideas
8. The process of discovering: recognizing mathematical ideas
9. The process of creating: generating mathematical ideas
10. The process of appreciating the place of mathematics: understanding its widespread applications to our daily activities
11. The process of problem solving: identifying, hypothesizing, interpreting, relating, evaluating, concluding

OBJECTIVES FOR GRADES 7-9 MATHEMATICS

It is important that grade 10 mathematics teachers become familiar with the mathematics programs in junior high, since the grade 10 programs outlined in this guide are intended to build on the skills possessed by students graduating from grade 9. Following are the objectives for grades 7-9 mathematics.

GRADE 7

As a result of the program, the student should be able:

1. To discuss the relative efficiency of the Hindu-Arabic number system compared to at least one other number system.
2. To identify the set of natural or counting numbers (N).
3. To apply the Property of One for multiplication.
4. To illustrate the set of whole numbers (W) as an extension of the set of natural numbers (N).
5. To apply the Property of Zero for addition and multiplication.
6. To illustrate the Closure Property as it applies to whole numbers.
7. To explain and to illustrate the structure of the Hindu-Arabic or decimal numeration system.
8. To rewrite numbers in expanded format, without using exponents, and vice versa.
9. To round off whole numbers.
10. To estimate, to calculate, and to check the solution to any problem involving the operations of addition, subtraction, multiplication, and division of whole numbers.
11. To estimate and to calculate the solution to number phrases involving several operations with or without brackets.
12. To demonstrate an understanding of the meaning of "power," "base," and "exponent."
13. To convert a power to a simple number and vice versa.
14. To apply the concept of exponents to expanded notation.
15. To perform simple calculations involving powers using the order of operations.
16. To solve and to check word problems pertaining to any of the preceding concepts or skills which involve whole numbers.
17. To express word phrases as number phrases or expressions and vice versa.
18. To evaluate open number phrases by substitution.
19. To solve simple one variable equations.
20. To translate simple word problems to equations and then to solve the equations.
21. To list the set of factors of a given number.
22. To list the set of multiples of a given number.
23. To determine the greatest common factor (GCF) of a set of composite numbers.
24. To determine the least common multiple (LCM) of a set of numbers.
25. To explain and to give examples of prime and composite numbers.

26. To write a composite number as a product of prime numbers.
27. To identify and to give examples of numbers that are relatively prime.
28. To demonstrate an understanding of a fraction as:
    a. a part of a whole,
    b. a part of a set,
    c. a comparison (ratio),
    d. a division.
29. To identify the numerator and denominator of a fraction.
30. To locate fractions on a number line and to identify the fractional equivalent of a point on a number line.
31. To identify proper and improper fractions.
32. To identify mixed fractions or mixed numbers.
33. To identify equivalent fractions.
34. To generate sets of equivalent fractions.
35. To change improper fractions to mixed fractions and vice versa.
36. To reduce fractions to lowest terms.
37. To compare fractions.
38. To add and to subtract:
    a. proper fractions,
    b. mixed fractions.
39. To multiply:
    a. proper fraction times whole number,
    b. mixed fraction times whole number,
    c. proper fraction times proper fraction,
    d. proper fraction times mixed fraction,
    e. mixed fraction times mixed fraction.
40. To find the multiplication inverse or reciprocal of any fraction or whole number.
41. To divide:
    a. proper fractions,
    b. mixed fractions.
42. To solve and to check word problems involving fractions.
43. To read decimal fractions to at least three decimal places.
44. To locate decimal fractions on a number line.
45. To write a decimal fraction in expanded notation without exponents.
46. To write a decimal fraction in expanded notation using only positive exponents.
47. To round off decimal fractions to the nearest whole number.
48. To round off decimal fractions to the nearest tenth, hundredth, or thousandth.
49. To change a fraction to a decimal fraction.
50. To rewrite a terminating decimal fraction as its fractional

equivalent.

51. To add and to subtract decimal fractions with any number of decimal places.
52. To multiply a decimal fraction by a power of ten (i.e., 10, 100, 1000, etc.).
53. To multiply a decimal fraction by a whole number other than a power of ten.
54. To multiply a decimal fraction by a decimal fraction involving any number of decimal places.
55. To divide a decimal fraction by a power of ten.
56. To divide a decimal fraction by a whole number other than a power of ten.
57. To divide a decimal fraction by a decimal fraction involving any number of decimal places.
58. To solve and to check word problems involving decimal fractions.
59. To demonstrate an understanding of ratio as a comparison of two numbers, using the notation:

$$a \text{ to } b \text{ or } a{:}b \text{ or } \frac{a}{b}$$

60. To set up a ratio between two quantities having the same unit of measure.
61. To set up a rate between two quantities having different units of measure.
62. To demonstrate an understanding of a proportion as equivalent ratios.
63. To apply and to solve problems relating to:
    a. scale drawings,
    b. map reading,
    c. unit pricing,
    d. miscellaneous problems.
64. To demonstrate an understanding of percent as a special ratio of a number compared to 100.
65. To change fractions to percent and vice versa.
66. To change decimal fractions to percent and vice versa.
67. To calculate a specific percentage of a given number.
68. To calculate what percentage one number is of another number.
69. To calculate provincial sales tax on a specified purchase.
70. To identify examples of line segments from the environment.
71. To estimate the length of a line segment in millimetres and in centimetres.
72. To use a metric tape or ruler to measure line segments to the nearest millimetre, centimetre, or metre.
73. To classify lines as being horizontal, vertical, or oblique.
74. To identify a pair of lines as being parallel or perpendicular.
75. To identify the lines of symmetry for segments.
76. To determine the perimeter of any polygon through measurement or through computation when given the lengths of the sides.
77. To determine the area of any square, rectangle, or parallelogram.
78. To identify the vertex and sides of an angle.
79. To name an angle using proper notation.
80. To measure the size of an angle less than 180°, using a protractor.
81. To draw any angle less than 180°, using a protractor.
82. To identify angles according to their size, as being acute, right, obtuse, or straight.
83. To identify the line of symmetry for any angle.
84. To use the proper notation to name a triangle.
85. To determine the perimeter of a triangle.
86. To measure each of the angles of a triangle using a protractor.
87. To classify triangles by sides and angles.
88. To identify the lines of symmetry for triangles.
89. To identify rotational or point symmetry.
90. To determine the area of a triangle.
91. To identify and to illustrate slides (translations), flips (reflections), turns (rotations), and combinations of these rigid motions.
92. To interpret and to construct a pictograph.
93. To interpret and to construct a bar graph.
94. To interpret and to construct a line graph.
95. To locate points and to identify coordinate pairs in the first quadrant.

GRADE 8

As a result of the program, the student should be able:

1. To demonstrate proficiency in performing operations with natural and whole numbers:
    a. by determining the next number in a given number sequence,
    b. by calculating the arithmetic mean and median for a set of data.
2. To test any number for divisibility by 2, 3, 4, . . . 10.
3. To demonstrate proficiency in the fundamentals:
    a. place value,

   b. expanded notation,
   c. rounding-off and estimation,
   d. prime and composite numbers,
   e. LCM and GCF for two or more whole numbers.

4. To identify the set of integers.
5. To identify the additive inverse (opposite) of any integer.
6. To add and to subtract integers.
7. To multiply and to divide integers.
8. To solve simple problems involving:
   a. income and expenditures,
   b. bank balances,
   c. temperatures,
   d. weights.
9. To change word phrases to open number phrases and vice versa.
10. To evaluate open number phrases by substitution in:
    a. one variable expressions,
    b. two variable expressions.
11. To add and to subtract like terms in one variable.
12. To simplify expressions involving both adding and subtracting.
13. To solve equations in one variable where the variable appears on only one side of the equation.
14. To translate a word problem into an equation and then to solve the equation.
15. To identify rational numbers.
16. To change a fraction to a terminating or a repeating decimal fraction by division or by using equivalent fractions.
17. To change a terminating decimal fraction to a fraction.
18. To change a repeating decimal fraction to a fraction using the "pattern of nines."
19. To perform the four basic operations using rational numbers.
20. To set up a ratio or a rate between two quantities.
21. To use proportions as equivalent ratios in solving problems.
22. To define percent as a special ratio.
23. To convert:
    a. fractions to percents and vice versa,
    b. terminating decimal fractions to percents and vice versa,
    c. repeating decimal fractions to percents and vice versa.
24. To round off percents to the nearest whole percent.
25. To estimate a percentage of a number.
26. To calculate a specific percentage of a given number.
27. To calculate what percentage one number is of another number.
28. To calculate a number when a percentage of it is known.
29. To apply the concepts of ratio and percentage relating to:
    a. provincial sales tax,
    b. commission,
    c. simple interest,
    d. discount,
    e. markup and markdown,
    f. scale drawings,
    g. recipes,
    h. mixtures.
30. To perform linear and angular measurement.
31. To determine the perimeter of any polygon.
32. To determine the area of any square, rectangle, and parallelogram.
33. To demonstrate an understanding of the concept of volume and to calculate the volume of rectangular solids (i.e., cubes and cuboids).
34. To solve problems related to perimeter, area, and volume.
35. To identify squares, rectangles, parallelograms, and kites.
36. To determine the lines of symmetry for various quadrilaterals.
37. To determine the rotational symmetry for various quadrilaterals.
38. To identify the centre, radius, diameter, arc, sector, and circumference of a circle.
39. To name related three-dimensional objects such as spheres, cylinders, and cones.
40. To draw a circle using a compass given a specific length for the radius or the diameter.
41. To determine the ratio of the circumference of a circle to its diameter and to establish the circumference formula.
42. To determine the area of a circle.
43. To determine the surface area of right circular cylinders:
    a. without ends,
    b. with ends.
44. To interpret circle graphs.
45. To construct circle graphs.
46. To identify and to illustrate slides (translations), flips (reflections), turns (rotations), and combinations of these rigid motions.
47. To identify parallel lines as slide images.

48. To identify perpendicular lines as a quarter turn about the point of intersection.
49. To demonstrate an understanding of the terminology associated with the coordinate plane.
50. To locate and plot points in all four quadrants and on the axes.

GRADE 9

The grade 9 course consists of 70% core and 30% option topics. Core materials are a continuation of grades 7 and 8 courses, while option topics are selected, three from five, according to the interests, abilities and future needs of the students.

CORE OBJECTIVES

As a result of the program, the student should be able:

1. To demonstrate proficiency in performing operations with whole numbers, fractions, and decimal fractions.
2. To explain and to illustrate the structure of the set of integers.
3. To explain and to illustrate concepts of:
    a. additive inverse (opposite),
    b. absolute value.
4. To demonstrate proficiency in performing the basic operations on integers.
5. To explain and to illustrate the meaning of rational numbers.
6. To change fractions to decimal fractions and vice versa:
    a. fractions to terminating or repeating decimal fractions,
    b. terminating decimal fractions to fractions,
    c. repeating decimal fractions to fractions.
7. To perform the four basic operations with rational numbers.
8. To solve problems relating to number operations.
9. To recognize irrational numbers.
10. To examine addition and multiplication of irrational numbers for closure.
11. To demonstrate an understanding of the definition of the set of real numbers as the union of rational and irrational numbers.
12. To demonstrate an understanding of the relationship between squares and square roots.
13. To find the exact square root of a perfect square.
14. To find the approximate square root of a natural number using:
    a. square root tables,
    b. a calculator.
15. To demonstrate an understanding of the concept of the Pythagorean relation.
16. To solve word problems involving the Pythagorean relation.
17. To use the Pythagorean relation to locate some irrational numbers on the number line.
18. To explain and to illustrate the algebraic concepts:
    a. variable,
    b. term,
    c. expression,
    d. numerical coefficient or coefficient.
19. To identify monomials, binomials, and trinomials.
20. To demonstrate an understanding of and to illustrate the meaning of the terms "power," "base," and "exponent."
21. To change a numerical power with natural number exponents where the base is either integral or rational to a simple real number.
22. To change a whole number into exponential form.
23. To demonstrate an understanding of the order of operations using exponents.
24. To evaluate open expressions in one and two variables using substitution.
25. To simplify open expressions involving addition and subtraction.
26. To multiply a monomial by a monomial.
27. To multiply a monomial by a polynomial.
28. To simplify open expressions involving addition, subtraction, and monomial multiplication.
29. To demonstrate proficiency in solving algebraic equations in one variable.
30. To translate word phrases into equivalent open expressions.
31. To solve algebraic word problems.
32. To perform the following basic constructions using only a compass and a straightedge:
    a. to construct the perpendicular bisector of a line segment,
    b. to bisect an angle,
    c. to copy an angle,
    d. to construct a perpendicular to a line through a point on the line,
    e. to construct a perpendicular to a line through a point not on the line.

33. To construct, without the aid of a protractor, angles of:
    a. 60° (hence 30°, 15°, 120°),
    b. 90° (hence 45°, $22\frac{1}{2}^\circ$, 135°),
    c. a composition of the above angles.
34. To copy triangles.
35. To construct triangles given the measures of:
    a. three sides,
    b. two sides and the included angle,
    c. two angles and any side,
    d. the hypotenuse and any side.
36. To identify and to demonstrate an understanding of the concept of opposite angles.
37. To demonstrate an understanding of the relationship between complementary angles and between supplementary angles.
38. To demonstrate an understanding of the concept of parallel lines.
39. To demonstrate an understanding of the relationship among angles created by parallel lines and a transversal:
    a. corresponding angles,
    b. interior or exterior alternate angles.

OPTION OBJECTIVES

Op. I Consumer Mathematics

40. To demonstrate an understanding of the following sources of income:
    a. earned:
        - salary (hourly, weekly, etc.)
        - professional fees,
        - self-employment,
        - commission.
    b. other:
        - welfare,
        - pensions,
        - family allowance,
        - interest,
        - unemployment insurance,
        - compensation.
41. To demonstrate an awareness of the major deductions which result in net income.
42. To demonstrate an awareness of the cost of living for:
    a. food,
    b. clothing,
    c. housing,
    d. transportation,
    f. insurance:
        - property,
        - automobile,
        - life,
        - crop,
    g. other.
43. To demonstrate the advantages, disadvantages, and the computations involving:
    a. cash vs. credit buying,
    b. borrowing money.

Op. II Statistics

44. To distinguish between a census study and a sampling of the population.
45. To demonstrate an understanding of random samples and stratified samples.
46. To identify errors which can be made in sampling the population.
47. To collect data and to determine:
    a. mean,
    b. median,
    c. mode.
48. To set up frequency distribution tables.
49. To interpret the data illustrated in the following graphs:
    a. bar graph,
    b. histogram,
    c. line graph,
    d. frequency polygon,
    e. circle graph.
50. To illustrate data by drawing labelled graphs of the preceding types.

Op. III Geometry

51. To identify congruent figures as having the same size and shape.
52. To relate congruence to the study of motion geometry and symmetry.
53. To use congruence notations.
54. To identify corresponding sides and corresponding angles of congruent figures.
55. To name polygons according to the number of sides.
56. To identify the properties of:
    a. triangles,

b. quadrilaterals,
c. other regular polygons.

57. To demonstrate an understanding of similarity as an enlargement or as a reduction of figures.
58. To use appropriate scale factors to enlarge or to reduce the size of an object.
59. To discover that similar figures have corresponding angles which are congruent.
60. To discover that similar figures have corresponding angles which are proportional (i.e., in the ratio given by the scale factor).
61. To discover the following by experimentation and direct measurement:
    a. the number of diagonals in a convex polygon,
    b. the angle sum for a polygon,
    c. the number of intersections of 2 lines, 3 lines, etc.,
    d. that in a circle, an angle at the centre is two times an angle at the circumference if they stand on the same arc,
    e. that an angle on a semi-circle is a right angle,
    f. that angles at the circumference subtended by the same arc are congruent,
    g. cyclic quadrilateral relationships.

Op. IV Algebra

62. To demonstrate an understanding of the concept that $m^0 = 1$ where $m \neq 0$.
63. To demonstrate an understanding of negative integral exponents.
64. To rewrite a given number in scientific notation and vice versa.
65. To multiply or divide two numbers using scientific notation.
66. To apply scientific notation and rounding off to estimate the product or quotient of very large or very small numbers.
67. To multiply a binomial by another binomial.
68. To divide a monomial by a monomial.
69. To divide a polynomial by a monomial.

Op. V Graphing

70. To demonstrate an understanding of the terminology associated with coordinate graphing including:
    a. coordinate axes,
    b. quadrants,
    c. coordinates (ordered pairs),
    d. origin.
71. To plot a point given its coordinates, and given a point, to write its coordinates.
72. To generate ordered pairs given the values of "x" using the equation $y = ax + b$ where "a" and "b" may be any real numbers.
73. To graph linear equations in the form $y = ax + b$, using a table of values.
74. To rearrange any linear equation into the form $y = ax + b$.
75. To graph two linear equations, on the same graph, in order to determine the point of intersection.

OBJECTIVES FOR GRADE 10 MATHEMATICS

There are three full credit course levels in grade 10 mathematics: 104, 101, and 100. Following are the objectives specific to each level.

MATHEMATICS 104 60% Core (C) - 40% Option (Op.)

C.I. ESTIMATION (1 module)

The student should be able:

1. To estimate the time required for a given task.
2. To make reasonable estimations in space.
3. To estimate arithmetic computations.
4. To make estimations involving money.

C.II. USE OF CALCULATORS (1/2 module)

The student should be able:

1. To enter numbers into a calculator so that the number displayed is identical to the original number.
2. To perform the basic operations of addition, subtraction, multiplication and division with whole numbers.
3. To perform the operations of addition, subtraction, multiplication and division with decimals.
4. To perform multiple operations using the calculator.

5. To perform percent calculations.

C.III. READING AND INTERPRETING GRAPHS AND TABLES (1/2 module)

The student should be able:

1. To read and to interpret information from tables.
2. To read and to interpret information from pictographs, bar, line, and circle graphs.
3. To answer questions using information attained from tables.

C.IV. MEASUREMENT: METRIC REVIEW (1/2 module)

A. Linear Measurement

The student should be able:

1. To identify the basic linear units (mm, cm, m, km).
2. To estimate various linear measurements.
3. To perform linear measurements using rulers and tape measures.
4. To convert from one basic linear unit to another.

B. Mass Measurement

The student should be able:

1. To identify the basic units of mass measurement (mg, g, kg, t).
2. To estimate various mass measurements.
3. To perform measurements of mass using scales.
4. To convert from one basic mass unit to another.

C. Capacity

The student should be able:

1. To identify the basic units for measuring capacity (mL, L).
2. To estimate the capacity of various containers.
3. To perform measurements of capacity.
4. To convert from one capacity unit to another.

C.V. TRANSPORTATION: THE AUTOMOBILE (1-1/2 modules)

A. Driver's Licence

The student should be able:

1. To discuss the completion of an application form for a driver's licence.
2. To interpret the information on a driver's licence.
3. To discuss the Manitoba merit-demerit point system.

B. Purchase and Insurance

The student should be able:

1. To identify sources of information for the purchase of an automobile.
2. To interpret vehicle sales ads.
3. To identify factors involved in the choice of a vehicle.
4. To discuss sales agreements and consumer protection laws involved in the purchase of a vehicle.
5. To examine the financing involved in the purchase of a vehicle.
6. To discuss Autopac Insurance coverage.

C. Maintenance

The student should be able:

1. To distinguish between "fixed" and "variable" costs.
2. To calculate the cost of a gasoline fill-up.
3. To determine the fuel economy in L/100 km.
4. To calculate the cost of specific repairs.
5. To calculate total "fixed" costs given the necessary information.

C.VI. EMPLOYMENT (1 module)

A. Forms

The student should be able:

1. To complete a standard job application form.
2. To discuss the completion of an application form for a social insurance number.
3. To complete a current TD-1 form.

B. Hourly Wages

The student should be able:

1. To calculate total wages over different periods of time (weekly, bi-weekly, semi-monthly, monthly) given a regular hourly rate.
2. To calculate the number of hours worked using a time card.
3. To compute total wages with overtime.

C. Pay Cheque Stubs

The student should be able:

1. To identify gross pay.
2. To identify compulsory deductions (e.g., CPP, UIC, Income Tax).
3. To identify miscellaneous deductions (e.g., charitable donations, bonds, pension, union dues).
4. To calculate net pay given gross pay and deductions.

C.VII. MONEY MANAGEMENT: BANKING (1 module)

A. Bank Accounts

The student should be able:

1. To discuss reasons for using banking institutions.
2. To identify the three main types of bank/trust/credit union accounts.
3. To simulate opening a specific bank account.

B. Deposit and Withdrawal Slips

The student should be able:

1. To complete a deposit slip.
2. To complete a withdrawal slip.

C. Cheques

The student should be able:

1. To write a cheque properly.
2. To distinguish between signing and endorsing a cheque.
3. To record the amount of each cheque in a journal or register.
4. To calculate the account balance after each cheque.

D. Bank Statements

The student should be able to read the information contained in an end-of-month statement and to answer questions pertaining to the given information.

The equivalent of 4 modules are to be selected from the following option topics.

Op.I. CLOTHING (1/2 module)

A. Purchases and Sales Tax

The student should be able:

1. To calculate the price of a single article of clothing given the price of multiple articles.
2. To calculate the price of several identical items of clothing given the price of one item.
3. To calculate total purchase cost.

B. Comparative Shopping

The student should be able:

1. To distinguish between the regular price and the sale price and to calculate the saving.

2. To calculate the sale price given the regular price and the rate of discount.
3. To distinguish between prices for the same or similar clothing merchandise from one store to another.

Op.II. EMPLOYMENT (1 module)

A. Applying for a Job

The student should be able:

1. To use the want ads to find a job.
2. To complete a standard written letter of application.
3. To complete a personal record (resumé).
4. To discuss factors involved in a job interview.
5. To identify several employee benefits.

B. Income from Employment

The student should be able:

1. To identify major forms of job payment including:
   a. hourly wages,
   b. salary,
   c. commission,
   d. salary plus commission,
   e. piecework,
   f. self-employment (farmers, fishers, trappers).
2. To calculate weekly or monthly earnings given a specified salary (contract) for a set time period.
3. To calculate earnings given a flat rate of commission.
4. To calculate earnings given a basic salary plus a commission.
5. To calculate earnings from piecework.

Op.III. LEISURE ACTIVITIES (1/2 module)

The student should be able to interpret and to complete mathematical exercises associated with a number of leisure activities.

Op.IV. APPLICATIONS AND METRIC MEASUREMENT (1 module)

A. Temperature

The student should know appropriate Celsius temperatures for a variety of situations.

B. Perimeter

The student should be able:

1. To demonstrate an understanding of the concept of perimeter.
2. To calculate the perimeter of a rectangle, square, and irregular shape given the appropriate dimensions.
3. To solve problems involving perimeter.

C. Area

The student should be able:

1. To demonstrate an understanding of the concept of area.
2. To calculate the area of a rectangle given the length and width.
3. To solve problems using the area of a rectangle.

D. Capacity and Volume

The student should be able:

1. To demonstrate an understanding of the concept of volume.
2. To calculate the internal volume of a rectangular solid.
3. To solve problems involving the volumes of rectangular solids.

Op.V. MONEY MANAGEMENT: BUDGETING (1 module)

A. Personal Budget

The student should be able:

1. To make a list of income and expenditures.
2. To complete a budget showing estimated income and expenditures.
3. To calculate the balance in a personal budget.

B. Family Budgets

The student should be able:

1. To make a list of regular expenses.
2. To complete a budget showing regular and discretionary expenditures.

Op.VI. RECREATIONAL MATHEMATICS (1/2 module)

The student should be able:

1. To solve mathematical puzzles.
2. To play mathematical games.
3. To participate in geometric activities.

Op.VII. GATHERING AND RECORDING DATA (1/2 module)

A. Gathering Data

The student should be able:

1. To list several kinds of data that are commonly gathered.
2. To distinguish between factual data and opinion.
3. To name several agencies that collect data and to state why they need data.
4. To list several methods of collecting data.
5. To collect data by counting and/or questionnaire.

B. Recording Data

The student should be able:

1. To organize data into tables with the headings: description, tally, and frequency.
2. To draw bar graphs and line graphs.

Op.VIII. TRANSPORTATION (1 module)

A. The Automobile--New/Used/Leased

The student should be able:

1. To identify reasons why people buy new/used/leased vehicles.
2. To compare cost over a 1, 3, or 5 year period for new/used/leased vehicles.

B. Planning a Trip Using Public Transportation

The student should be able:

1. To interpret bus schedules.
2. To interpret train or airline schedules.
3. To plan a route using appropriate bus/train/airline schedules.

C. Planning a Trip by Auto, Train, Airplane, Bus

The student should be able:

1. To interpret a highway map.
2. To calculate distances between given cities on a map.
3. To approximate travel time for given distances.
4. To approximate total travel costs for a particular trip.

MATHEMATICS 101 50% Core (C) - 50% Option (Op.)

C.I. ARITHMETIC (1/2 module)

A. Whole Numbers

The student should be able to demonstrate proficiency in the fundamental arithmetic of addition, subtraction, multiplication and division with whole numbers.

B. Integers

The student should be able to demonstrate proficiency in the addition and subtraction of integers.

C. Decimal Fractions

The student should be able to demonstrate proficiency in the fundamental operations of addition, subtraction, multiplication and division with positive decimal fractions.

D. Fractions

The student should be able to convert fractions to decimal fractions.

E. Multiplication and Division of Signed Numbers

The student should be able to multiply and to divide signed numbers (both whole numbers and decimal fractions).

F. Order of Operation

The student should be able to evaluate numeric expressions involving several operations.

G. Percent

The student should be able:

1. To identify percent as the numerator of a fraction with 100 as the denominator.
2. To convert percents to decimal fractions and vice versa.
3. To determine the value of a stated percent of a given number.

H. Estimation

The student should be able:

1. To round off given numbers to stated place values.
2. To round off numbers and then to perform the appropriate arithmetic operations.

C.II. CONSUMER MATHEMATICS (1-1/2 modules)

A. Income

The student should be able:

1. To calculate gross wages earned, given:
    a. hours worked and rate per hour,
    b. hours worked including overtime and the rate per hour for weekly and daily time periods,
    c. rules for late and early arrival, time cards, and rates of pay.
2. To calculate net pay given gross pay and deductions.
3. To calculate new salary given present salary and percent rate of increase.
4. To calculate the percent rate of increase, given the amount of increase and the present salary.

B. Budgeting

The student should be able:

1. To calculate the amount of individual expenditures given the income and the percent rate for each expenditure.

2. To calculate the percent rate of each expenditure given the income and the amount of each expenditure.
3. To prepare a cash budget form given:
    a. income and expenditures,
    b. income and percent rates of expenditures.

C. Consumer Purchasing

The student should be able:

1. To calculate provincial sales tax.
2. To calculate unit costs and to make comparisons.
3. To calculate the amount of savings given the sale price and the regular price of an article.
4. To calculate the amount of savings and the sale price given the percent rate of discount and the regular price.
5. To calculate the percent rate off, given the regular price and the amount saved.

D. Banking

The student should be able:

1. To identify and to compare accounts offered by financial institutions, including chequing, chequing-savings, and savings accounts.
2. To write a cheque.
3. To calculate balances on chequing accounts given the amount of deposits and cheques.
4. To update a cheque book journal or register, given cheques, deposits, and service charges.
5. To reconcile a bank statement.

C.III. GEOMETRY (1-1/2 modules)

A. Perimeter

The student should be able:

1. To demonstrate an understanding of the con-

cept of perimeter as a measure of the distance around.
2. To state the common metric units of perimeter (mm, cm, m, km) and to convert from one unit of linear measure to another.
3. To determine the perimeter of a polygon when given the lengths of the sides and when measurement is required.
4. To solve word problems involving perimeter.

B. Area

The student should be able:

1. To demonstrate an understanding of the concept of area as the number of unit squares on a surface.
2. To state the common metric units of area (mm^2, cm^2, m^2, km^2) and to convert from one area measure to another.
3. To determine the area of a square, rectangle, parallelogram, triangle, trapezoid, and rhombus through computations or through measurement using squared paper.
4. To solve word problems involving area.

C. Volume

The student should be able:

1. To demonstrate an understanding of the concept of volume as the number of unit cubes in a given solid.
2. To see that in some cases (cube, rectangular solid) the volume is equal to the area of the base times the height.
3. To state the common metric units of volume (mm^3, cm^3, m^3, km^3) and to convert from one volume measure to another.
4. To determine the volumes of rectangular solids, when given the lengths of the sides and through measurement.
5. To solve word problems involving volume.

D. Application of Perimeter, Area, and Volume

The student should be able:

1. To solve word problems involving perimeter, area, and volume by recognizing the type of measurement required and then making the appropriate calculations.
2. To recognize and to distinguish between the units of measurement for perimeter, area, and volume.
3. To determine the missing dimension given the perimeter, area, or volume and the other dimension(s).

E. Capacity

The student should be able:

1. To recognize the common metric units of capacity (mL, L).
2. To convert from one capacity unit to the other.

F. Mass

The student should be able:

1. To recognize and to state the common metric units of mass (mg, g, kg, t).
2. To convert from one mass unit to another.

G. Identification of Angles

The student should be able to identify the sides and the vertex of an angle, and to name an angle.

H. Angular Measurement

The student should be able:

1. To recognize the units of angular measurement as degrees or fractions of a revolution.
2. To measure angles from 0° to 360° in either revolutions or degrees (to the nearest degree).
3. To draw with a protractor any angle whose measure is less than or equal to 360° or 1 revolution.
4. To identify angles according to their size as being acute, right, obtuse, straight, or reflex.

C.IV. ALGEBRA (1-1/2 modules)

A. Exponents

The student should be able:

1. To explain and to illustrate the meaning of the algebraic terms:
   a. base,
   b. exponent,
   c. power,
   d. coefficient.
2. To convert a power with a natural number exponent and a rational base into a simple real number.
3. To convert an integer into exponential form.
4. To understand the meaning of square root and to recognize the symbol for square root.
5. To find the square root of numbers using either calculators or square root tables.
6. To evaluate given formulae by substitution.

B. Polynomial Operations

The student should be able:

1. To identify and to collect like terms.
2. To add or to subtract algebraic expressions.
3. To multiply and to divide monomials by integers and decimal fractions.
4. To multiply and to divide algebraic expressions by integers and decimal fractions.
5. To simplify algebraic expressions involving addition, subtraction, and multiplication.

C. Solving Equations

The student should be able:

1. To write an equation when given the solution.
2. To solve linear algebraic equations in one variable.
3. To solve simple quadratic equations of the form $ax^2=c$ where "a" and "c" are constants with the same sign and $a \neq 0$.
4. To express word phrases as algebraic expressions and vice versa.
5. To translate word problems into algebraic equations and vice versa.

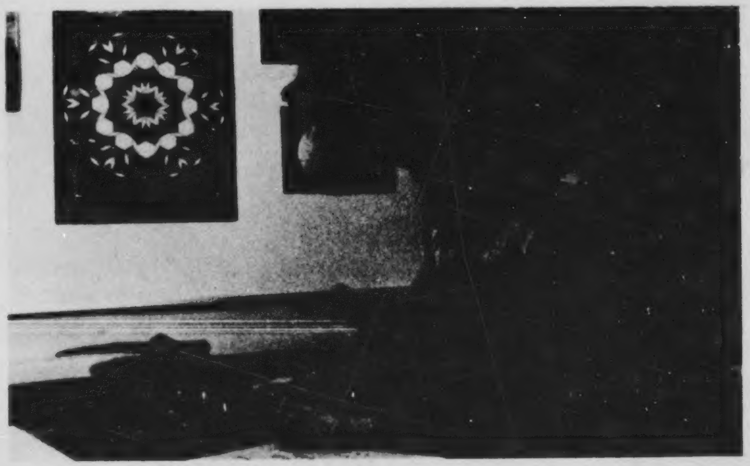
Teachers/students are to select the equivalent of 5 modules from the following option topics.

CALCULATOR MATHEMATICS

Op.1. OPERATIONS OF AND APPLICATIONS FOR CALCULATORS (1/2 module)

The student should be able:

1. To perform on a calculator the basic operations of addition, subtraction, multiplication, and division.
2. To solve problems using the correct order of operations.
3. To convert common fractions to decimal fractions.
4. To perform calculations involving percent.
5. To perform operations involving square roots and reciprocals.
6. To use the memory function.
7. To recognize and to interpret unusual displays.



BUSINESS MATHEMATICS OPTIONS

Op.II. BASIC SKILLS IN BUSINESS MATHEMATICS (I module)

A. Rapid Calculation

The student should be able:

1. To identify the place value of each digit from a thousandth to a million.
2. To read and to write numbers between thousandths and millions.
3. To add a column of numbers.
4. To add, to subtract, to multiply, and to divide whole numbers.
5. To add, to subtract, to multiply, and to divide decimal fractions.
6. To multiply by 0.001, 0.01, 0.1, 10, 100, 1000 by moving the decimal point.
7. To divide by 0.001, 0.01, 0.1, 10, 100, 1000 by moving the decimal point.
8. To convert one type of fractional number to another type:
   a. common fraction to decimal fraction,
   b. common fraction to percent rate,
   c. decimal fraction to percent rate,
   d. percent rate to decimal fraction.
9. To find the percentage of a number given the percent rate and the number.
10. To calculate the price of a quantity of articles if the price of one is known.
11. To calculate Manitoba sales tax given the percent rate of provincial sales tax and the price of an article or articles.

B. Pricing

The student should be able:

1. To round off numbers involving dollars and cents.
2. To calculate the cost of a specific number of articles given the price for a quantity purchase.

C. Making Change

The student should be able to make change given the cost of the article and the amount offered in payment.

Op.III. DISCOUNT CALCULATIONS (I module)

A. Single Trade Discount

The student should be able:

1. To demonstrate an understanding of the meaning of these words: list price, retail price, retailer, wholesaler, manufacturer, trade discount, rate of trade discount, invoice price, net price.
2. To calculate the trade discount given the list price and the trade discount rate.
3. To find the net price given the rate of trade discount and the list price.
4. To calculate the rate of trade discount given the amount of trade discount and the list price.
5. To calculate the list price given the rate and amount of trade discount.

B. Series of Trade Discounts

The student should be able:

1. To explain the meaning of the term "series of discounts" or "discount series."
2. To calculate the net price given the list price and a series of trade discount rates.
3. To find the amount of trade discount given the list price and a series of trade discount rates.
4. To find a single discount equivalent to a series of trade discounts.
5. To find the net price, using a discount table, given a series of trade discounts and the list price.

C. Cash Discount, Cash Price, and Invoices

The student should be able:

1. To distinguish between trade discounts and cash discounts.
2. To explain the meaning of and notation for "terms of sale."
3. To find the last date of payment for cash discount eligibility given the date of the invoice and the terms of sale.
4. To find the cash price given the list price, terms of sale, date of invoice, and payment date.
5. To calculate the total cost given the list price, trade discount, date of invoice, terms of sale, date of payment, and transportation charges.
6. To explain f.o.b. terms and to calculate total charges involving f.o.b. amounts.
7. To answer questions regarding a given invoice form.
8. To calculate the total cost given an invoice form.

Op.IV. MERCHANDISE AND SERVICE BUSINESS CALCULATIONS (1 module)

A. Markon (Markup) and Markdown

The student should be able:

1. To demonstrate an understanding of the following terminology as applied to business: markon, selling price, and markdown.
2. To find the amount of markon and the selling price, given the percent rate of markon (based on the cost) and the cost.
3. To calculate the percent rate of markon based on cost, given the cost and the selling price.
4. To calculate the percent rate of markon based on the selling price.
5. To calculate the amount of markdown, given the original selling price and the reduced price (new selling price).
6. To calculate the percent rate of markdown given the original selling price and the reduced price.

B. Income Statement

The student should be able:

1. To calculate total revenues given a variety of sources.
2. To calculate net revenue given sales returns and/or allowances.
3. To calculate total expenses given a variety of expenditures.
4. To explain the difference between revenues, expenses, and profits.
5. To prepare an income statement given revenues, sales returns and allowances, and expenses.

VOCATIONAL OPTIONS

Op.V. MEASUREMENT AND ESTIMATION (1 module)

A. Measuring Instruments

The student should be able:

1. To read ruler scales to the nearest millimetre.
2. To read metric vernier calipers.
3. To read metric micrometers.

B. Reading Diagrams

The student should be able:

1. To read measurements from diagrams.
2. To read measurements from floor plans.
3. To read measurements from site plans.

C. Reading Meters

The student should be able:

1. To read natural gas and hydro meters.
2. To read voltmeters, ohmmeters, and/or ammeters.
3. To read common graduated scales.

D. Cost Calculations

The student should be able:

1. To determine area and to calculate the total cost given the cost per unit area.
2. To determine volume and to calculate the total cost given the cost per unit volume.
3. To solve problems involving time and money calculations.
4. To estimate the quantity of materials needed for particular jobs.

E. Mass Calculations

The student should be able:

1. To determine the value of the unknown given two of the three quantities: mass per unit area, total mass, and total area.
2. To determine the value of the unknown given two of the three quantities: mass per unit volume, total mass, and total volume.

Op.VI. RATIO AND PROPORTION (1 module)

A. Ratio

The student should be able:

1. To explain the meaning of the word "ratio" and to illustrate the four standard ways of writing ratios.
2. To reduce ratios to lowest terms.
3. To reduce ratios so that a numerator or denominator of one (1) results.
4. To express the rate of automobile fuel consumption in L/100 km.

B. Application of Ratios

The student should be able to apply ratios to solving problems that occur in various areas of business, trade, or industry.

C. Proportion

The student should be able:

1. To explain the meaning of the word "proportion."
2. To solve a proportion for an unknown quantity when the other three are known.

D. Direct Proportions

The student should be able:

1. To explain what is meant by the term "directly proportional."
2. To set up a direct proportion between two variables and, given sufficient information, to solve for an unknown quantity.
3. To determine the size of a real object given the scale and the scale drawing, and to find the size for a scale drawing given the scale and size of the real object.
4. To apply proportions to problems involving similar triangles.

E. Inverse Proportions

The student should be able:

1. To explain what is meant by the term "inversely proportional."
2. To solve problems involving inverse proportions.

Op.VII. PERCENT CALCULATIONS (1 module)

The student should be able:

1. To differentiate the three types of percent problems.
2. To determine the amount given the base and the percent rate.
3. To determine the percent rate given the base and the amount.
4. To determine the base given an amount and its percent rate.
5. To solve percent problems which occur in various areas of business, trade, or industry.

GENERAL MATHEMATICS OPTIONS

Op.VIII. GRAPHING (1 module)

The student should be able:

1. To construct and to interpret a bar graph.
2. To interpret various kinds of bar graphs.
3. To construct and to interpret a single line graph.
4. To interpret various types of line graphs.
5. To construct and to interpret a circle graph.
6. To construct and to interpret a pictograph.
7. To recognize distortion in graphs.

Op.IX. ESTIMATION (1/2 module)

The student should be able:

1. To estimate the time required for a given task.
2. To make reasonable spatial estimations.
3. To estimate arithmetic computations.
4. To estimate the total cost, including sales tax, and change associated with consumer buying.

Op.X. MATHEMATICAL EXPERIMENTS (1/2 module)

The student should be able:

1. To perform a mathematical experiment given written instructions.
2. To state observations and conclusions concerning the concept(s) involved in the mathematical experiment.

Op.XI. SPATIAL RELATIONS (1/2 module)

A. Motion Geometry

The student should be able:

1. To identify and to perform the basic transformations (reflection, rotation, translation) on geometric figures (both linear and curved).
2. To identify lines of symmetry of figures and the reflections which will carry a figure onto itself.
3. To identify the centre(s) of rotational symmetry and the order of rotational symmetry.

B. Pattern and Perspective

The student should be able:

1. To lay out a pattern from which each of the following can be folded:
   a. a cube,
   b. a tetrahedron,
   c. a rectangular solid.
2. To decide whether a given layout can be folded to give a cube, tetrahedron, or rectangular solid.
3. To make perspective drawings of:
   a. a cube,
   b. a rectangular solid,
   c. a prism,
   d. a mixture of the above.

C. Illusion

The student should be exposed to optical illusions.

Op.XII. STATISTICS (1/2 module)

A. Introduction

The student should be able:

1. To define statistics.
2. To discuss the role of statistics in modern society.
3. To recognize the need to organize raw data.

B. Sampling

The student should be able:

1. To distinguish between a census and a sampling of the population.
2. To use various methods for collecting data.
3. To describe ways in which the collection of data may be biased.

C. Organizing Data

The student should be able:

1. To calculate the following measures of central tendency or "averages" using ungrouped data:
   a. the arithmetic mean,
   b. the median,
   c. the mode.
2. To select the measure of central tendency most representative of the group of data.
3. To determine the range of ungrouped data.
4. To organize sample data using a frequency distribution table.
5. To calculate the mean for grouped data.
6. To display data using a histogram.

D. Projects

The student should be able to complete a project which involves collecting the data, organizing and/or graphing the data, and displaying the data in a histogram.

MATHEMATICS 100 85% Core (C) - 15% Option (Op.)

C.I. EQUATIONS (1 module)

The student should be able:

1. To solve equations of the following types:
   a. linear,
   b. equations which can be reduced to linear form having numerical coefficients,
   c. literal equations,
   d. simple quadratic equations.
2. To evaluate formulae by substitution.
3. To reduce equations to the quadratic form $ax^2 + bx + c = 0$ where a, b, c are integers and $a \neq 0$.
4. To translate word phrases into algebraic expressions.
5. To solve problems which involve translation into an equation.

C.II. POLYNOMIAL OPERATIONS (1 module)

The student should be able:

1. To identify and to classify algebraic terms.
2. To combine like terms.
3. To perform algebraic subtraction.
4. To use the basic exponent laws as applied to terms.
5. To perform general algebraic multiplication.
6. To add or to subtract products of binomials.
7. To use Pascal's triangle in binomial expansions.
8. To perform general algebraic division.
9. To express the result of an algebraic division in alternate forms.

C.III. FACTORIZATION AND RATIONAL EXPRESSIONS (1-1/2 modules)

The student should be able:

1. To demonstrate an understanding of the term "factoring."
2. To reduce a rational expression with factored numerator and denominator.
3. To reduce a rational expression using common factoring.
4. To factor a difference of squares.
5. To reduce a rational expression using difference of squares and/or common factoring techniques.
6. To factor trinomials.
7. To reduce rational expressions using common, difference of squares, and/or trinomial factoring techniques.
8. To multiply and to divide rational expressions.
9. To add and to subtract rational expressions.

C.IV. RADICALS (1 module)

The student should be able:

1. To find square roots of non-negative rational numbers using either a calculator or a table.
2. To demonstrate an understanding of the Pythagorean relation by solving application problems.
3. To simplify algebraic square roots.
4. To perform basic operations on terms involving radicals.
5. To rationalize the numerator or the denominator of a radical expression.
6. To solve equations involving radicals.

C.V. SIMULTANEOUS EQUATIONS (1/2 module)

The student should be able:

1. To determine graphically whether a pair of simultaneous first degree equations are inconsistent (parallel lines), consistent and dependent (the same line), or consistent and independent (lines that intersect in exactly one point).

2. To solve a system of two first degree equations in two unknowns using the addition-subtraction method.
3. To translate problems into pairs of simultaneous first degree equations.

C.VI. COORDINATE GEOMETRY (1/2 module)

The student should be able:

1. To demonstrate an understanding of the terminology associated with the coordinate plane.
2. To locate and to plot points in the four quadrants and on the axes.
3. To graph a straight line given its equation in the form $y - mx + b$ where "m" and "b" are real numbers.
4. To define the slope and the y- intercept of a line.
5. To determine the value of the slope and the y-intercept:
   a. from the equation of the line in the form $y = mx + b$,
   b. from the graph.
6. To write the equation of a straight line in the form $y = mx + b$:
   a. given the slope and y- intercept,
   b. given the graph from which the slope and y- intercept can be obtained.
7. To write the equation of a vertical line in the form $x = k$, given the graph.

C.VII. DESCRIPTIVE - DEDUCTIVE GEOMETRY (3 modules)

A. Points, Lines, Planes, and Angles

The student should be able:

1. To name a segment or angle using standard notations.
2. To estimate and to measure:
   a. segments to the nearest millimetre or centimetre,
   b. angles to the nearest degree.
3. To demonstrate an understanding of the terms collinear, non-collinear, and between.
4. To demonstrate an understanding of the definitions and terminology associated with angles.
5. To determine angle measure from a diagram using angle addition and/or subtraction.
6. To demonstrate an understanding of and an ability to recognize in diagrams the following types of angles or angle pairs:
   a. acute, obtuse, and right,
   b. vertical,
   c. adjacent,
   d. supplementary and complementary,
   e. linear pairs.
7. To demonstrate an understanding of congruence as applied to segments and angles.

B. Introduction to Proof

The student should be able:

1. To recognize and to draw conclusions using the property of equality for real numbers.
2. To recognize and to draw conclusions using the transitive property of congruence for both segments and angles.

C. Using Certain Basic Geometric Properties to Develop Proofs

The student should be able:

1. To discover the congruence of vertical angles and to use it in proof questions.
2. To demonstrate an understanding of the definition of bisection as relating to segments and angles and to use it in proofs.
3. To demonstrate an understanding of the definition of perpendicularity relating to lines and segments by using this definition in proofs.
4. To discover the congruence of the base angles of an isosceles triangle and to use it in proofs.
5. To demonstrate an understanding of the Segment Addition and Subtraction Properties by using them in proofs.

6. To demonstrate an understanding of the interior of an angle and the Angle Addition and Subtraction Properties and to use these in writing proofs.
7. To use the concepts of linear pair, supplementary angles and complementary angles in proofs.

D. Using the Congruence of Triangles to Develop Proofs

The student should be able:

1. To demonstrate an understanding of a congruence correspondence between two triangles.
2. To postulate the SAS congruence property, to use it to prove the congruence of triangles, and to name the remaining corresponding congruent side and angles of the triangles.
3. To postulate the SSS congruence property, to prove triangles congruent, and to identify corresponding congruent angles.
4. To postulate the ASA congruence property, to use it to prove the congruence of triangles, and to name the remaining corresponding congruent angle and sides of the triangle.

5. To write the converse of a given statement having the form "if p, then g."
6. To prove the converse of the following statement and to use it in proofs: "If a triangle is isosceles, then two of its angles are congruent."

GEOMETRY OPTION

Select one of the following geometry options:

Op.I. VECTORS (1/2 module)

A. Introduction

The student should be able:

1. To distinguish between vector and scalar quantities.
2. To interpret and to use compass directions.
3. To name and to represent on diagrams vectors using:
   a. magnitude and direction,
   b. geometric and algebraic representations,
   c. ordered pairs.

B. Equal Vectors

The student should be able to define and to identify equal vectors.

C. Vector Addition

The student should be able to add two vectors.

D. Vector Multiplication

The student should be able:

1. To multiply a vector by a number.
2. To multiply a vector by a scalar (with units).

E. Vector Problems

The student should be able to use vector addition and scale drawings to solve relevant vector problems.

Op.II. ORIENTEERING (1/2 module)

A. Orienteering on the School Grounds

The student should be able:

1. To demonstrate an understanding of the magnetic compass.
2. To find the bearing of a visible object.
3. To find the direction of a given bearing.
4. To measure distances using the number of paces (double step) required for 100 metres.
5. To find a station, given a starting point, a bearing, and a distance.
6. To follow an orienteering trail from one station to the next, given the bearing and distance between each station.

B. Orienteering with a Map

The student should be able:

1. To read a contour map and to be able to interpret all the aids and obstacles necessary to follow an orienteering line.
2. To demonstrate an understanding of magnetic declination, back bearings and cross bearings.
3. To demonstrate the ability to detour around an object or obstacle and to get back on the same bearing as before the detour.
4. To follow an orienteering route marked out on a map, with a separate sheet identifying the bearing and the distance between each station.
5. To follow an orienteering route with only the stations marked on a map.

CONSUMER MATHEMATICS

Select one of the following consumer mathematics options.

Op.III. MONEY MANAGEMENT (1 module)

A. Income from Employment

The student should be able:

1. To identify and to solve problems associated with types of income.

2. To calculate an individual's net periodic pay given the appropriate rates/amounts of deductions.
3. To complete a basic income tax return.

B. Banking and Interest

The student should be able:

1. To perform calculations involving simple interest.
2. To perform calculations involving compound interest.

C. Credit System

The student should be able to perform calculations involving credit buying.

Op.IV. CONSUMER STATISTICS (1 module)

The student should be able:

1. To organize a given set of data derived from a discrete variable into an ungrouped frequency table.
2. To classify given ungrouped discrete data and to make a grouped frequency table.
3. To name the exact limits of a given class interval, and to group a given set of data derived from a continuous variable into a table.
4. To construct a histogram and a frequency polygon to illustrate a given set of data.
5. To identify sources of bias related to a given graph's structure.
6. To calculate the value of the measures of central tendency (averages) including:
   a. mean,
   b. median,
   c. mode.
7. To choose the "average" which could be used to greatest advantage for a given purpose.
8. To determine the range of a given set of data.
9. To devise a sampling technique which will avoid obvious sources of bias.
10. To pose an experimental statistical problem, to design the data-gathering methods, to gather the data, and to present it appropriately using methods developed in this module.

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IMPLEMENTATION

RATIONALE

This guide contains the goals generally thought desirable for the cognitive development of grade 10 mathematics students. But the core objectives for each grade level are prescriptive rather than exhaustive, and teachers are encouraged to consider these in relation to their particular classroom situations when formulating learning outcomes. Material in this guide takes into account the skills possessed by grade 9 graduates and the needs that they have and will develop. For this reason, it is necessary for grade 10 teachers to become familiar with the mathematics programs in both junior high and grade 11.

Mathematics as a formal structure must be built upon a foundation of concrete experiences and applications. According to developmental learning theory, students progress through three stages of learning: concrete (objects), semi-concrete (pictures), and abstract (symbols). Care must be taken to articulate the grade 10 program with patterns of development established in the K-6, 7-9 guides where emphasis is placed on the use of concrete materials. It may not be necessary or possible in every case to begin at the concrete level. If the material makes first-hand experience impractical, or if previous experiences have developed the underlying concepts, then initial activities may be at the semi-concrete or abstract levels. However, teachers are reminded that not all students in grade 10 can function totally at an abstract level. As students progress or as topics, concepts and processes require higher levels of cognitive development, the time devoted to concrete and semi-concrete activities decreases while the time spent in more abstract activities increases.

Basic skill development in the grade 10 student should be balanced with an understanding of the structure and vocabulary of mathematics. To promote this balance, the guide de-emphasizes terminology, stresses understanding and recommends appropriate practice. Further, it urges that topics, once developed, be reviewed and maintained.

Teachers are encouraged to employ a variety of teaching techniques incorporating visual aids, concrete materials, a number of different texts, puzzles, games and activities. The games, activities and suggested materials in this guide can be supplemented with those that teachers consider appropriate to the needs of their students.

If the program is to be relevant to the present needs of students and, at the same time, provide the foundations for future learning, then mathematics should be correlated with other subjects. A meaningful correlation will go far towards achieving a deep and continuing interest in mathematics and an appreciation for its power and usefulness. The nature and means of this correlation will be determined at the school level through the specific programs being offered to the students. Teachers are encouraged to be aware of ways of correlating and integrating mathematics with other subjects.

GENERAL GOALS FOR 04, 01, 00 MATHEMATICS

In identifying the specific goals of the mathematics program, one must consider the present needs of individuals and society. The student goals upon which this guide is based are identified by course below.

04 Mathematics

1. To develop mathematical skills in the context of practical, everyday situations;
2. To realize the importance of mathematics for all consumers;
3. To develop facility with estimations;
4. To develop further understanding and facility with measurement;
5. To observe, to collect, to interpret, and to display data graphically;
6. To perform arithmetic calculations accurately using a calculator;
7. To become more proficient in problem solving related to work and leisure;
8. To participate in recreational or leisure mathematics activities;

9. To increase confidence in estimating answers and in using mathematical skills to solve everyday problems;
10. To develop further the skills of reading, listening and following directions.

01 Mathematics

1. To develop an awareness of the usefulness of mathematics;
2. To increase understanding of the concepts, and to develop mastery of basic arithmetic skills using whole numbers integers and decimal fractions through the solution of practical problems in the consumer, business, trade and vocational areas;
3. To become more proficient in mental arithmetic and rapid calculation exercises;
4. To increase comprehension in the problem solving process;
5. To perform reasonable estimations in terms of both geometric and numerical activities;
6. To develop further understanding and facility with metric measurement and language;
7. To increase consumer awareness with reference to both the market and work place;
8. To develop an understanding and a mastery of basic algebraic skills;
9. To increase inductive and deductive reasoning and other thinking skills;
10. To cultivate a positive attitude towards mathematics.

00 Mathematics

1. To develop a proficiency in problem solving;
2. To understand and to master basic algebra skills;
3. To observe and to identify geometric relationships involving two-dimensional figures;
4. To develop skills in deductive and inductive reasoning;
5. To interpret graphs and to display data graphically;
6. To realize that mathematics is important for all consumers;
7. To develop positive attitudes towards, and curiosity about, mathematics;
8. To read mathematics with understanding;
9. To develop the capability for post-secondary training in mathematics.

THE GUIDE

The program outlined in this guide is a description of a set of basic skills for grade 10 mathematics students. Work in subsequent grades relies on the mastery of these basic skills and an understanding of the concepts developed in this guide. Where necessary, enrichment activities should be included.

1. Terminology

Module - section of a mathematics course involving approximately 10% of the allotted time. A full credit mathematics course would consist of 10 modules, some compulsory, others chosen from a prepared list of optional modules. For a non-semestered class, each module could be subdivided as follows:

14 periods for learning new concepts
4 periods for review, testing, remediation and/or enrichment

Any reference to periods in this guide refers to 40-minute class periods.

Core - compulsory modules which all students registered in a particular course are required to study. Core modules are designated "C."

Option - module(s) selected from a prepared list. A specified number of these are to be completed in order to fulfill course requirements. Option modules are designated "Op."

2. *Sequencing*

The grade 10 mathematics courses outlined in this guide consist of core and option modules in a topical arrangement. A preferred order has not been identified.

Rather, the order to be followed should be determined by the teacher in accordance with available texts. Several options are possible:

a. Some teachers may prefer to complete all the core units before beginning option topics.
b. Other teachers may wish to integrate topics from core and option modules.
c. Others may wish to alternate core and option modules.
d. Still others may choose to intersperse particular option topics within core topics, using, for example, one period every week or two.

3. *Time Allotment*

As credit courses, Mathematics 104, 101, and 100 each require 110-120 hours of instruction time. All schools/school divisions are expected to ensure that sufficient time is allocated to provide for the development of the objectives in individual mathematics programs.

ROLE OF TEXTBOOKS

Independent of any one textbook, the guide allows for flexibility in scope and approach. It provides a framework, but encourages teachers to consult outside materials in order that programs be enriched or modified to meet student needs. Teachers need access to and familiarity with various resources in order to accommodate the different ability and interest levels of students, to select appropriate content and methodology, and to have at hand various exercises, activities, and games for remediation, enrichment and problem solving.

USE OF CALCULATORS

"The world is presently experiencing an explosion of electronic technology which is threatening to render obsolete much of what we teach in school mathematics. Electronic calculators, unknown in the early 70's, are so prevalent that they can be found in nearly every home or office. Because of their low cost and wide spread use in society, calculators have the potential of reshaping the mathematics curriculum in our schools. School practices generally lag far behind societal changes and the utilization of calculators in school is no exception. While on-the-job and in-the-home use of calculators to perform complex computations is nearly standard, schools have yet to incorporate calculators into the mathematics and science curricula. This must change."[1]

In high school (9-12), the use of calculators is strongly recommended. They are useful for computations involving powers and roots, consumer mathematics, and problem solving, and should replace mathematics tables for both logarithmic and trigonometric functions. As most computational skills have been introduced and developed in grades K-8, the emphasis in high school should be directed towards concept development and application.

In today's society, the use of calculators has increased the significance of two interrelated mathematics skills: checking reasonableness of answers and estimation. The many students who tend to believe that calculators inevitably give the correct answer should learn to realize that this confidence is unwarranted, since incorrect input operations produce wrong and unreasonable results. To develop the students' awareness of reasonable results, estimation skills should be an integral part of all mathematics programs.

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[1] Charlotte L. Wheatley, "Calculator Use in the Middle Grades," *School Science and Mathematics*, 80, No. F (1980), p. 620.

It is important that students be taught when and how to use the calculator properly. Total reliance on it is not the objective. Students must realize that at times it is more expedient to use paper and pencil calculations or mental arithmetic and that they still should use paper and pencil to record the steps in problem solving.

It is also important to explore ways in which calculators may be used in the acquisition of mathematical concepts, in the study of numerical patterns, and in the acquisition of ideas.

"Calculators . . . should be used in imaginative ways for exploring, discovering and developing mathematical concepts and not merely for checking computational values or for drill and practice."[2]

Students should be instructed in the types of calculators available for purchase, the functions that different types perform, and the functions necessary for each course level. Students should be advised against the purchase of an expensive programmable calculator, since the technology is developing so rapidly that calculators which are purchased this year could be outdated and much less expensive next year.

USE OF COMPUTERS

Wherever possible, mathematics teachers and students should use computers to full advantage. Students could be exposed to the computer as a tool for use in developing the concepts of logical thinking through algorithms, and in investigating mathematical concepts which can be made more meaningful through use of a computer. Topics could include powers and roots, wages and payroll, area and volume, compound interest, statistical values, solution of simultaneous equations, loans and mortgages, problem solving, etc.

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[2] NCTM, *An Agenda for Action: Recommendations for School Mathematics of the 1980's*, April, 1980.

Computer literacy should be an essential outcome of contemporary education. Students should acquire an understanding of the versatility and limitations of the computer by using it to solve mathematics problems. In particular, problem solving, simulations, data retrieval tasks, and drill/practice programs would best serve to integrate the computer and mathematics.

EVALUATION

Evaluation of student achievement and program effectiveness must be an integral, ongoing part of the teaching-learning process. Evaluation should provide information regarding:

1. *Diagnosis and remediation*. The purpose of diagnosis is to identify a student's strengths and weaknesses. In the case of weaknesses, the cause of the problem may then be identified and appropriate remediation provided by the teacher.

2. *Student growth in mathematical skills and understanding*. This is an integral part of a developmental program. Because growth is continuous, evaluation must also be

continuous. Teachers should assess a student's speed and accuracy, understanding of concepts and processes, ability in applying skills, and should use evaluation to encourage further student attainment in mathematics.

3. Learning strategies. Results should be used to judge the effectiveness of the instructional process: program of experiences, materials of instruction and strategies of teaching.

4. Curriculum modification. Modifications should be ongoing. Therefore, continuing feedback from the schools regarding the appropriateness of the objectives and the degree of implementation of the current program is essential.

Testing is a vital component of evaluation. Tests may be structured to evaluate a specific concept, a series of interrelated concepts, or all the concepts found in a year's work. Classroom teachers should design and use cumulative tests so that students maintain proficiency in previously learned concepts and skills.

Students in grade 10 should be faced with some form of cumulative testing (be it cumulative unit tests or final examinations) involving the objectives outlined in this guide. Both oral and written tests could be used. Every effort should be made to have such tests go beyond the lower level cognitive skills and to examine higher level cognitive objectives.

Evaluation is far more than paper and pencil testing. Teachers are reminded that evaluation of student achievement may take several additional forms such as daily observation, interviews or conferences with individual students, class discussions, and student reports and projects related to definitive and open-ended problem solving situations.

SPECIAL NEEDS STUDENTS

Students with special needs are those with handicaps, whether physical, cultural, emotional or intellectual, those who have been identified as slower learners, and those who are gifted or talented. Students in each of these groups require programs that are adapted to their needs. Suggestions for modifying the program are discussed.

Handicapped Students

The educational philosophy currently favoured for dealing with handicapped students is normalization, a philosophy that, in many cases, has resulted in the mainstreaming of these individuals. However, a key concept of normalization is the "least restrictive placement" which implies that all special needs students should not be mainstreamed.

Integration into the regular program will depend upon thorough identification of and attention to the needs of special students. These needs can be assigned to one of the following four categories:

Physical	- general health disability, deafness or hearing impairment, blindness or visual impairment, orthopedic handicap.
Cultural	- inadequate language development or experiences for growth.
Emotional	- problems with personal relationships, mental illness.
Intellectual	- underdeveloped cognitive ability, especially in the areas of conceptualization and abstract thinking.

Students with needs in any of these categories may be mainstreamed, depending on the degree of the need, but there are at least three conditions which may be considered essential for successful mainstreaming.

1. Good communication must exist among the student, classmates and teacher.
2. The student must be socially acceptable to the class.
3. The student's intelligence and background must enable him/her to adapt to the program content and demands, to whatever extent it is modified.

Slower Learners

Not all special needs students will have significant intellectual or physical handicaps. Indeed, the majority of students labelled "slow learners" come from cultural or socioeconomic environments that differ from the norm. Whether such students simply learn more slowly or whether their potential for learning is inherently limited is a subject of considerable academic debate. The progress of students who have been identified as slower learners in the regular program should be monitored closely. If adaptation of the regular program does not meet their needs, then these students should be transferred to another program.

Modifying the Program

Student background and experience--including lifestyle, interest, motivation--are the most important factors in determining the extent to which a handicapped student or slower learner may be accommodated in the regular program. Consideration of these factors,as well as of a student's cognitive ability, his or her learning strengths and weaknesses, should determine the selection of appropriate teaching strategies and materials. Some specific suggestions for modifying the program to suit the needs of these students are listed below.

1. Choose those options in this guide whose content is appropriate to student background/interest, or use new content commensurate with the overall objectives.

2. Select materials appropriate to the student's reading level. If there is a foreign language problem, use E.S.L. teaching materials developed by the Department of Education.

3. Be aware of the student's preferred learning modality; i.e., Is he/she a better visual, auditory or tactile/kinesthetic learner? Use the stronger modality.

4. Teach from the concrete to the abstract. Use a "life skills" approach.

5. Enhance the student's self-concept by providing activities which will ensure recognizable achievement.

6. Be sure the student knows exactly what is required. Give clear instructions to minimize misunderstanding. Teach one skill or objective at a time. Use only one sensory approach at a time to avoid student confusion.

7. Use community resources. Develop activities so that students see the practical value and application of what they learn in class.

8. Incorporate suggestions from the 04 course into your program.

9. Consider transferring the student into another program if the previous adjustments prove ineffective.

Gifted or Talented Students

Gifted students in mathematics may show one or more of the following characteristics:

- outstanding performance in most academic subjects;
- prolific reading and ability to read and understand mathematics textbooks and articles;
- preference for independent work with little teacher direction;
- curiosity towards abstract concepts;
- assumption of responsibility.

Since the Mathematics 100 outline identifies the minimum program for students, and since gifted students can attain the course objectives in less time than other students, teachers of mathematically gifted students are faced with the challenge of enriching the program with stimulating and worthwhile activities. Suggestions include the following.

1. Stimulating Interest

   - Provide a wide variety of materials and resources, such as The Mathematics Teacher (NCTM) and other professional journals/magazines.

   - Emphasize reading in mathematics.

   - Encourage the pursuit of problems of interest to the student.

   - Use mathematics clubs to maximum advantage.

   - Organize seminars at which students can present papers and special projects for ultimate submission to journals and newspapers.

   - Encourage peer tutoring.

2. Meaningful Activity

   - Enrich the program rather than accelerate the students.
   - Provide independent activities.
   - Encourage entry in mathematics contests and competitions.
   - Promote computer use in mathematics.
   - Encourage mathematics research.
   - Encourage students to apply their mathematics to fields such as engineering, finance, meteorology, science, urban planning, computer science, economics, statistics, etc. Applications of mathematics are difficult in that they require knowledge of the application area in addition to the ability to apply pure mathematics in unfamiliar situations.

3. Additional Skills

   - Provide higher level problem solving, research, reading and writing skills activities than those required for regular programs.
   - Emphasize the development of students' manipulative intellectual skills.

IMPLEMENTATION

Mathematics Development-Writing Committees developed course outlines for Mathematics 104, 101, 100 during the 1979-80 school year. Pilot programs were established in September 1980, and interim documents were distributed the following year.

Effective September 1982, Mathematics 104, 100, 100 become the only first year high school courses to satisfy the student's mathematics requirement for graduation. Mathematics 102 and 103 will no longer be recognized for credit purposes, except for the student who repeats the program during the 1982-83 school year.

The following table outlines curriculum development for 04, 01, 00 Mathematics.

Grade Level	School Year					
	1979-80	1980-81	1981-82	1982-83	1983-84	1984-85
10	Develop	Pilot	Interim	Final		
11		Develop	Pilot	Interim	Final	
12			Develop	Pilot	Interim	Final

mathematics 104

MATHEMATICS 104

INTRODUCTION

Basic skills in mathematics are necessary for functioning in today's society. The 04 program concentrates on the application of mathematical skills and concepts to everyday situations. Skills covered include information processing, reasoning, computation, and problem solving in areas encountered by the student at home, at work and at leisure. Students intending to pursue post-secondary education should be advised against taking this program.

GENERAL GOALS OF THE 04 PROGRAM

The student goals upon which this program is based are as follows:

1. To develop mathematical skills in the context of practical, everyday situations;
2. To realize the importance of mathematics for all consumers;
3. To develop facility with estimation;
4. To develop further understanding and facility with measurement;
5. To observe, to collect, to interpret, and to display data graphically;
6. To perform arithmetic calculations accurately using a calculator;
7. To become more proficient in problem solving related to work and leisure;
8. To participate in recreational or leisure mathematics activities.
9. To increase confidence in estimating answers and in using mathematical skills to solve everyday problems;
10. To develop further the skills of reading, listening and following directions.

04 MATHEMATICS STUDENTS

There are several categories of students taking the 04 sequence of mathematics courses:

1. Those choosing Mathematics 104, 204, 304 for credit towards regular high school graduation.
2. Those taking a complete 04 program in which mathematics is integrated with other subjects including Work Experience.
3. Those taking a complete 04 program in which mathematics is taken as a separate course.

For this reason, the competence of students in 04 courses varies greatly. Although it is intended that teachers follow the objectives of the course, it is also recognized that the difficulty of the examples and the complexity of the problems chosen will vary from class to class and from student to student. Each course is organized into modules which allow either a whole class or an individualized approach.

FEATURES OF THE 04 COURSES

1. The 04 courses approach mathematics from the point of view of "everyday life" situations.

2. Estimation and the use of calculators are considered to be particularly important to the students of these courses. Thus, they are included at the beginning of the 104 course. It is intended that calculators be employed wherever appropriate. Note that the need to concentrate on the use of calculators may be greatly decreased in future years if students are taught this information in previous grades.

3. Major course themes/topics include: accommodation, business management, clothing, employment, food, leisure activities, money management, recreational mathematics and transportation. Specific mathematical topics dealt with include measurement and geometry, statistics and graphing, and algebraic applications.

4. The teaching of basic arithmetic skills (i.e., the use of whole numbers, decimals, simple fractions, metric units and percentages) is to be incorporated into the courses as needed. All students should tackle problems and applications to be found in everyday life and should become aware of situations that require them to apply mathematical skills with accuracy. It is not expected that students will have reached a predetermined skill level before doing applications.

5. The individual needs of students vary greatly. Teachers may need to provide remediation for some students together with simple applications or, alternatively, may find that some students can solve more complex problems.

STRUCTURE OF THE 04 COURSES

The following table gives an outline of the time and emphasis allotted to the topics identified for Mathematics 104, 204, 304, and specifies the year that they are to be introduced. The numbers represent the number of modules.

TOPICS	MATH 104		MATH 204		MATH 304	
	Core	Options	Core	Options	Core	Options
Accommodation			1½	1	1	½
Algebraic Applications					½	½
Business Management			1			½
Clothing		½				
Computer Awareness						1
Employment	1	1				
Estimation/Mental Arithmetic	1		½		½	
Food			½	½		
Graphing/Statistics	½	½	½	½		1
Insurance					1	
Leisure Activities		½		½		
Measurement/Geometry	½	1	1½	1		
Money Management/Records	1	1	½	1	1	1
Probability						½
Ratio, Rate and Percent					2	
Recreational Mathematics		½		½		
Taxation				1		1
Transportation	1½	1				
Use of Calculators	½					
	6	6	6	6	6	6

Each full module accounts for 14 periods of new work in a non-semestered system. This should allow approximately 4 periods per module for review and testing. Ten modules comprise a full credit. Core modules are compulsory, and each 04 course will have the equivalent of 6 core modules.

Schools offering Work Experience as part of their program may need to modify the suggested time allotment because parts of some modules may have already been covered. It is quite conceivable that the Math 104 course for students on extended Work Experience would consist of only 8 modules.

THE 104 COURSE

Core Topics (Complete each of the following)

			Page References
C.I.	Estimation	(1 module)	45
C.II.	Use of Calculators	(½ module)	49
C.III.	Reading and Interpreting Graphs and Tables	(½ module)	54
C.IV.	Measurement: Metric Review	(½ module)	60
C.V.	Transportation: The Automobile	(1½ modules)	62
C.VI.	Employment	(1 module)	68
C.VII.	Money Management: Banking	(1 module)	70

Option Topics (Select the equivalent of 4 modules)

Op.I.	Clothing	(½ module)	73
Op.II.	Employment	(1 module)	75
Op.III.	Leisure Activities	(½ module)	78
Op.IV.	Applications of Metric Measurement	(1 module)	79
Op.V.	Money Management: Budgeting	(1 module)	82
Op.VI.	Recreational Mathematics	(½ module)	85
Op.VII.	Gathering and Recording Data	(½ module)	87
Op.VIII.	Transportation	(1 module)	88

TEACHING SUGGESTIONS

1. The unit on estimation should be covered, in whole or in part, at the beginning of the year and the concepts used and reinforced throughout the program.
2. The unit on the use of calculators should be taken early in the course so that realistic problems involving numbers other than whole numbers may be used.
3. The content outlined for Math 104 is arranged by topic/theme and, apart from the previous two recommendations, the sequence to be followed is to be determined by the teacher/student.

C.I. ESTIMATION (1 module)

The ability to determine whether a solution to a mathematical problem is reasonable or realistic is a vital mathematical skill in a society advocating the use of calculators. Students have a tendency to believe that the number that appears on a calculator after an operation has been performed is correct. It is important for them to realize that such a belief is unwarranted since incorrect input operations will produce wrong and unreasonable results. In order to develop a student awareness of reasonable answers, teachers must focus on estimation. Increased attention should be placed on estimation of time, of space, of cost, and of number.

Since estimation is difficult and is possible only after concepts are understood, the purpose of this unit is to teach the concepts associated with estimation. Teachers should use uncomplicated examples to establish the ideas which then should be integrated into the remainder of this and other 04 math courses.

The suggested method of handling this unit is as follows:

1) Spend an initial period of time, say 3 or 4 class periods, giving students experience in estimation. Concentrate on estimation involving time and space rather than arithmetic operations.

2) Integrate all remaining time throughout the remainder of the course.

Discussion of the parameters necessary for making reasonable time estimates should be encouraged. A major aim of the type of exercise found below is to encourage students to read or listen and then follow directions.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to estimate the time required for a given task;	Students should learn what they can hope to accomplish in a specified period of time. Practical examples involving check-out time at a supermarket, time to shingle a roof, etc., could be employed. Wise use of time, particularly short periods of time, should be encouraged. Students should learn to maximize productivity in the classroom, thereby making time outside class available for leisure, social, or job-related activities. Students should be required to estimate and to measure time, and then to compare the two results in situations such as the following: 1) walking from Point A to Point B. Individual students could be asked to estimate the time to go from A to B and return. Then, they could be timed by other members of the class when they actually walk the predetermined route.

OBJECTIVES

NOTES TO TEACHERS

2) completing a specified number of arithmetic exercises using a calculator or paper and pencil techniques. Students should estimate the time it will take them to finish the exercises, then do the calculations as accurately and as quickly as possible. Teachers should stress the need for accuracy in such activities as speed tests, accuracy tests, mental arithmetic, etc.

3) using public transportation.

4) perceiving the passage of time under different conditions. Students could be asked to estimate time both while actively participating in an activity and while acting as a spectator. Comparisons of the two estimates should be made. Students should realize that time seems to pass more quickly when the mind is occupied.

2. to make reasonable spatial estimations;

Students should estimate, record the estimate, measure, record the measure, and then compare the estimate with the measure in physical measurement situations.

Students could concentrate on a number of the following:

1) metric measures of length (Pacing, handspans, etc., could be used to assist students in making reasonable estimates for distance. Such estimates should begin with the smaller units such as mm and cm and then should proceed to larger units such as m and km.)

2) measures of perimeter, area and volume (Different shaped figures should be used. For example, have students compare the volume of a tall rectangular solid with that of a more compact one.)

3) height and mass (weight) calculations

4) shoe sizes

5) the number of sheets of paper in a given stack; the number of pennies required to make a stack 4 cm high

6) the number of students capable of standing in a square metre

7) the number of objects, people or things in a group

8) capacity

It is not enough to say, "estimate." Teachers must give systematic and regular effort to the skill of making spatial estimations. The following steps are suggested:

OBJECTIVES	NOTES TO TEACHERS
	Step 1: Have students relate objects to each other through the use of appropriate comparison words such as bigger, longer, shorter, heavier, thicker, taller, etc.
	Step 2: Have students practice estimation and direct measurement on objects that are familiar and present.
	Step 3: Shift the emphasis, and have students estimate objects which are not in view. Follow this with actual measurement of the objects.
	Step 4: Finally, reverse the process and encourage students to give examples of objects which are approximately of a given length, area, volume, size, etc.
	Games could be used to reinforce the process of estimation.
	Note: Many of the ideas in this section should be integrated with the measurement portions of this and other courses that the students may be taking. See C.IV Measurement: Metric Review; C.V Transportation; Op.I Clothing; Op.III Leisure Activities; Op.IV Applications of Metric Measurement; and Op.VIII Transportation.
3. to estimate arithmetic computations;	If students are to estimate arithmetic computations, they should have experience in: 1) betweeness and the analysis of number relations (Teachers are advised to keep this analysis at a very general level. For example, students must realize that 2.4 is between 2 and 3 and slightly closer to 2.) 2) rounding numbers (Students should be able to round numbers to the nearest 100, 10, 1, $\frac{1}{10}$, and $\frac{1}{100}$.) Example: Round 234.9 to the nearest hundred. Answer: 200. 3) multiplying/dividing by powers and multiples of 10 4) multiplying/dividing by values between 0 and 1 (Students tend to believe that multiplying increases the value whereas dividing decreases the initial value. They must realize that when multiplying or dividing by numbers between 0 and 1 the opposite is true.) 5) mental calculations

OBJECTIVES	NOTES TO TEACHERS
	Note: Students traditionally have had difficulty with the estimation of arithmetic computations. It appears that paper and pencil algorithms discourage estimation in arithmetic because of the desire for exact answers. Teachers are encouraged to initiate practice sessions that focus on estimation to the point of excluding paper and pencil calculations. Students should be urged to use mental arithmetic in conjunction with their calculators.
4. to make estimations involving money.	Examples could include total cost, sales tax and change associated with a series of purchases. The complexity of calculations will be determined by student ability.

C.II. USE OF CALCULATORS (½ module)

One of the reasons for teaching mathematics is to enable students to solve problems in the real world, particularly those arising from their own experiences. A calculator is a useful tool for helping them do this. Not only can students solve problems more quickly with a calculator than with paper and pencil, but they can also do more difficult problems so long as they understand which operation is required. Students whose background in arithmetic is weak can gain many benefits from the use of the calculator.

Since the calculator can give answers that are wrong and unreasonable if input operations are incorrect, it is essential that students first estimate the answer to a problem, then calculate the solution, and finally compare the two answers. Emphasis should be placed on estimation, on properly written solutions, and on the reasonableness of answers. Refer to C.I Estimation.

Advantages to Using the Calculator:

a. Generating student interest in mathematics
b. Building student self-confidence in computational skills
c. Providing students with immediate feedback
d. Providing students with opportunity to solve more problems
e. Allowing students to solve problems that previously have been too time consuming to be done with pencil and paper
f. Allowing students to solve more difficult problems
g. Providing "survival" computational information for those students unable to develop the basic concepts of addition, subtraction, multiplication or division.

Suggestions:

a. Students should spend time at the beginning of the school term learning to use their calculators properly. Throughout the term, calculator use should be reinforced with daily/weekly practice sessions. Teachers are to encourage the use of the calculator throughout this and other 04 mathematics courses.
b. Students should be encouraged to do mental arithmetic, using arithmetic speed tests, oral drills, etc.
c. Schools are not expected to purchase classroom sets of hand-held calculators, but should have a limited number available for the use of those students unable to purchase or borrow one. Students of Mathematics 104 should be encouraged to purchase calculators capable of performing the four fundamental arithmetic operations (addition, subtraction, multiplication and division), square roots and percent calculations. They should be discouraged from purchasing complicated scientific calculators for which they will have little or no immediate use.
d. Because "entry" errors are so easily made, calculator users must be able to judge the correctness of a solution. Good estimation and rounding skills are vital to this judging procedure. See C.I Estimation.

OBJECTIVES

The student should be able:

1. to enter numbers into a calculator so that the number displayed is identical to the original number;
2. to perform the basic operations of addition, subtraction, multiplication and division with whole numbers;

NOTES TO TEACHERS

Review as necessary. Both oral and written exercises should be employed.

One way to introduce a computation skill with the calculator is to present examples of the computation (addition, subtraction, multiplication or division) on a sheet, overhead projector or blackboard along with the answers. Give students the task of checking the answers.

Example: Use a calculator to check examples such as:

$$\begin{array}{r} 43 \\ +\ 37 \\ \hline 80 \end{array} \qquad \begin{array}{r} 69 \\ +\ 54 \\ \hline 125 \end{array}$$

The above procedure should be followed for the remaining operations of subtraction, multiplication and division.

Students should learn to operate a calculator from both oral and written instructions. Oral drills should be used with all four arithmetic operations. Dictate the question that students are expected to solve on their calculators.

Once students understand the correct procedure and have developed some confidence and competence in using their calculators, they can be given drill sheets without answers.

Note: The order of procedure will depend on the calculator model being used.

Special Notes on Division

When a division operation is performed and the answer is not a whole number, the calculator displays digits to the right of the decimal point that should be rounded off. For example, when 349 ÷ 33 is entered into the calculator, 10.575757 is the answer. Students must realize that this is approximate and probably should be rounded to, say, 10.58.

In division the student must know which number to enter first.

OBJECTIVES

3. to perform the operations of addition, subtraction, multiplication and division with decimals;

4. to perform multiple operations using the calculator;

NOTES TO TEACHERS

Examples:

1) Divide 16 by 5
2) 16 divided by 5
3) Divide 5 into 16
4) 16 ÷ 5
5) $\frac{16}{5}$
6) $5\overline{)16}$

Inverse operations could be used to check answers.

Most calculators have a floating decimal point. Thus, except that the decimal point must be entered, all four arithmetic operations with decimals are exactly the same as with whole numbers.

Rounding off is of great importance when working with decimals.

Note: Activities involving money may serve as a very practical introduction to decimal fractions.

The sequence of difficulty should proceed from:

1) chain addition or subtraction, to
2) addition and subtraction (credit-debit), to
3) chain multiplication or division, to
4) grocery bills, time cards, etc.

Multiple operations are involved in calculating a grocery bill. Some operations should be performed first and a record kept of the findings. After these primary computations are completed, the others follow, depending on the particular problem.

Examples:

1) Find the cost of the following grocery bill.

4 kg apples @ $1.25/kg = ______ (operation is x)
25 kg beef @ $3.25/kg = ______ (operation is x)
3 tins soup @ 5/$1.73 = ______ (first operation is ÷ to find cost of 1; second operation is x to find cost of 3)
TOTAL: = ______ (final operation is +)

Operations included in this example are: x, ÷, and +.

2) Perform the following additions. Simplify.

a) 11 hr. 16 min.
27 hr. 39 min.
Answer: 38 hr. 55 min.

b) 3 hr. 21 min. 14 sec.
4 hr. 45 min. 19 sec.
Answer: 8 hr. 6 min. 33 sec.

Note: Students should be exposed to games involving calculations, such as the following:

1) Estimation game - 2 players. Two players each have 30 seconds to estimate a product or quotient of 2-, 3-, or 4-digit numbers. The player with the closer estimate wins a point. The first player with 10 points wins.

2) Target game. Players alternate pressing a number and ⊗ to reach a target number. The first player to come within 1 of the target number wins.

3) Game of 21 - 2 players. On each turn a player can push only one of the three keys 1, 2, or 3 along with the ⊕ key. The first player to reach 21 is the winner. (Have students determine the winning strategy.)

4) Game of 50 - 2 players. Same as above except using the keys 1, 2, 3, 4, 5 and 6.

Recreational mathematics can be used to encourage calculator usage.

Examples could include:

a) When the numerals on a hand-held calculator are inverted, they form some of the letters of the alphabet. Examples include:

1 → I
3 → E
4 → h
5 → s
6 → g
7 → L
9 → b
0 → O

Using these letters, many words and phrases can be formed which are the result of pre-planned calculations. Examples are available from calculator books/booklets. However, students should be encouraged to create their own examples.

b) A number has 1 for all its digits. If the number is divisible by both 7 and 13 what is the smallest number it could possibly be?

c) Determine the missing digits.

$$\begin{array}{r} 3\,5\bigcirc 4 \\ 7\,1\,9\bigcirc \\ \bigcirc 6\,2\,8 \\ 6\bigcirc 3\,9 \\ \hline 22\,1\,2\,2 \end{array}$$

Exercises could involve addition, subtraction, multiplication, division or combinations of these operations.

5. to perform percent calculations.

Limit examples to determining a specified percent of a number. Applications could deal with reductions and sales tax.

Examples:

1) Tanya saved 25% of the cost of a $55.00 skirt. How much did she save?

Answer: $55.00 x 25% = $13.75.

2) How much sales tax must be paid on a sale of $19.59 if the sales tax rate is 7%?

Answer: $19.59 x 7% = $1.37.

References:

G. Immerzeel and E. Ockenga, The Calculator Books, Books 1, 2 (Menlo Park, Calif.: Addison-Wesley, 1979).

J. N. C. Sharp, The Calculator Work Box (Don Mills, Ont.: Addison-Wesley Canada, 1977).

G. Vervoort and D. Mason, Calculator Activities for the Classroom, Books 1, 2, 3 (Toronto: Copp Clark Pitman, 1979).

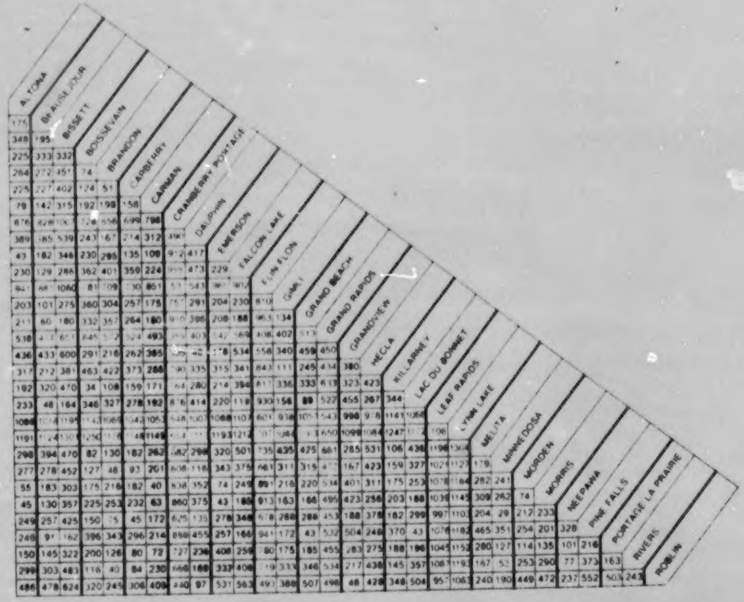
C.III. READING AND INTERPRETING GRAPHS AND TABLES (½ module)

Reading and interpreting graphs/tables should be treated as a distinct and separate topic from preparing graphs/tables. Work with graphs and charts should be incorporated into other sections of this course. The types of tables, graphs and charts included in this unit are not exhaustive and others appropriate to particular classes may be used in addition to or instead of those shown. Teachers should encourage students to make use of tables, graphs and schedules available locally.

OBJECTIVES	NOTES TO TEACHERS
The student should be able: 1. to read and to interpret information from tables;	The following is part of the chart on the official Manitoba Highway Map for 1982. Similar charts are found on other road maps and atlases. Five (5) complimentary copies of new or current Manitoba Highway Maps are available for classroom use from: Travel Manitoba, 7th Floor, 155 Carlton Street, Winnipeg, Manitoba R3C 0V8.

Example:

Find the distance from Beausejour to Falcon Lake.


ALTONA
BEAUSEJOUR
BISSETT
BOISSEVAIN
BRANDON
CARBERRY
CARMAN
CRANBERRY PORTAGE
DAUPHIN
EMERSON
FALCON LAKE
FLIN FLON
GIMLI
GRAND BEACH
GRAND RAPIDS
GRANDVIEW
HECLA
KILLARNEY
LAC DU BONNET
LEAF RAPIDS
LYNN LAKE
MELITA
MINNEDOSA
MORDEN
MORRIS
NEEPAWA
PINE FALLS
PORTAGE LA PRAIRIE
RIVERS
ROBLIN

OBJECTIVES

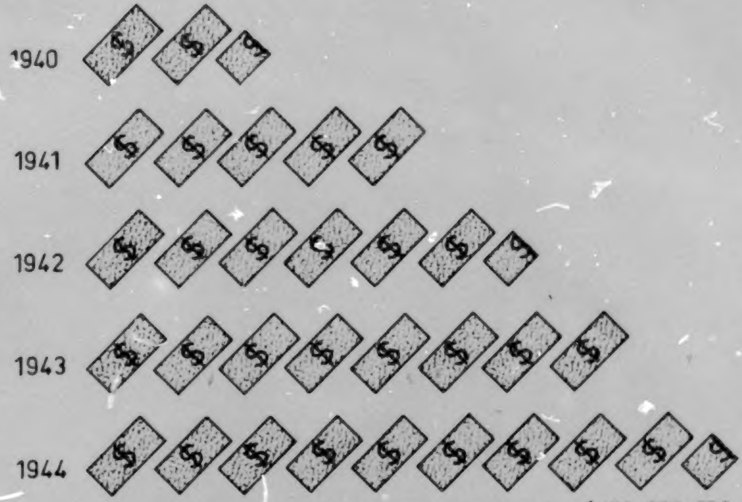
2. to read and to interpret information from pictographs, bar, line, and circle graphs;

NOTES TO TEACHERS

1) Pictographs show comparison for more than one time period.

Example:

AMOUNT OF PAPER MONEY IN CIRCULATION
IN CANADA, 1940-1944



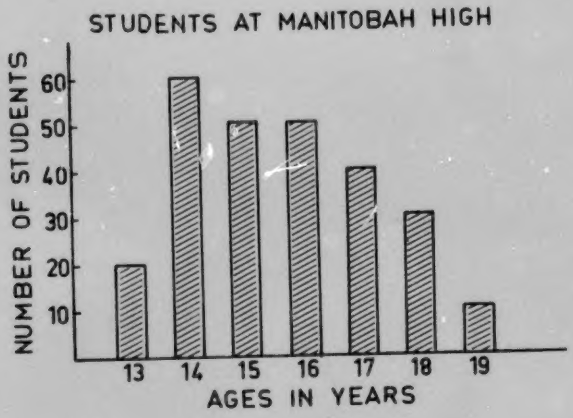
Each symbol represents $100 000 000

Use the above graph to answer the following questions:

a) How much money was in circulation in 1940? in 1943?
b) In what year was the least amount of money circulated?
c) How much did the amount of money in circulation increase from 1940 to 1941? From 1942 to 1944?

2) Records which show comparison between statistical information - often sizes of quantities - for more than one time period are often illustrated by bar graphs.

Example:



Use the above graph to answer the following questions:

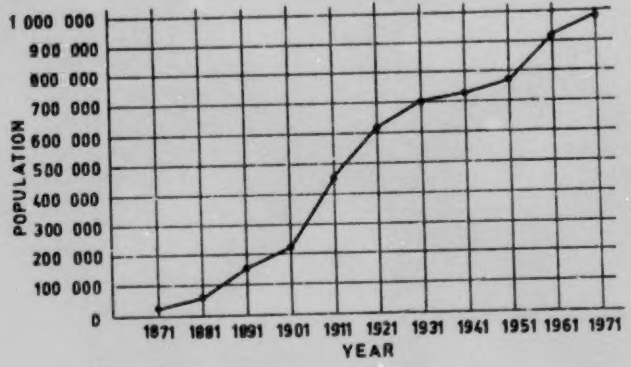
a) How many students are 14 years old? 17 years old?
b) Which age group has the most students?
c) How many students are there altogether?
d) How many students are over 15 years of age?
e) How many students are younger than you?

OBJECTIVES | NOTES TO TEACHERS

3) Line graphs are often used to show change over a period of time. The vertical scale usually indicates the change while the horizontal scale shows the time.

Example:

THE POPULATION OF MANITOBA 1871 - 1971



Use the above graph to answer the following questions:

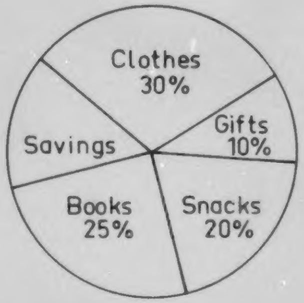
a) What was the population in 1961? in 1901?
b) By how much did the population increase between 1901 and 1961?
c) In which decade was the increase in population the greatest?

OBJECTIVES | NOTES TO TEACHERS

4) Circle graphs show the relation of the parts to the whole and to each other.

Example:

ANNUAL PERCENTAGE OF
PERSONAL ALLOWANCE EXPENDITURES



Use the above graph to answer the following question:

a) If the allowance totals $260 a year, how much money goes into savings?
   i) $15
   ii) $26
   iii) $32
   iv) $39

OBJECTIVES

3. to answer questions using information attained from tables.

NOTES TO TEACHERS

Examples:

1) Telephone Rates

Use the charts found in the current telephone directory to answer the following:

a) What is the cost of a 3-minute call from Winnipeg to The Pas at 8:00 p.m. on a Wednesday if it is a customer-dialed call?
b) What is the cost of a 10-minute call from Portage la Prairie to Winnipeg on a Monday at 10:00 a.m. if it is an operator-handled, person-to-person call?

2) Transportation Schedules

Copies of airline, bus and train schedules may be used.

3) 100 Ways to Save Energy and Money in the Home, 1978.

This booklet is available from Energy, Mines and Resources Canada and contains a number of charts, tables and graphs. Consider the following example:

How Much Do You Pay For a Bath?

Depth of Water	Temperature of Water		
Centimetres	Warm 50°C	Hot 60°C	Very hot 70°C
	cents	cents	cents
10	6	8	10
13	8	11	13
15	10	13	15
18	12	15	18
20	13	17	20

Additional examples could include postal rates, sports or weather data, from such sources as Statistics Canada Publications, Guiness Book of Records, old catalogues, magazines, etc.

Teachers may wish to incorporate some of the concepts found in Op. VII Gathering and Recording Data.

C.IV. MEASUREMENT: METRIC REVIEW (½ module)

It is expected that students will already have been exposed to the metric system in previous grades. This module should serve as a "refresher" course in estimating and measuring with metric units. Note: The approach throughout this review should be to "think metric." No conversions from or to the Imperial system should be attempted.

After students have gained practical experience in measuring, they should become practiced in converting one basic metric unit to another.

Throughout work in all three areas (linear--mass--capacity), the "guess" and then the "actual measurement" approach should be taken. Students can review the basic metric units by doing several measurement problems. Refer to C.I Estimation.

Note: Appropriate units must be chosen and attention paid to the degree of accuracy needed in various cases.

Examples: km ---- distance
kg ---- mass
L ---- capacity

OBJECTIVES	NOTES TO TEACHERS
A. LINEAR MEASUREMENT	
The student should be able:	
1. to identify the basic linear units (mm, cm, m, km);	
2. to estimate various linear measurements;	Examples: 1) How much is your handspan? 2) How long is your thumb? 3) How wide is your desk?
3. to perform linear measurements using rulers and tape measures;	
4. to convert from one basic linear unit to another.	Conversions should be limited to mm, cm, m and/or km.

OBJECTIVES	NOTES TO TEACHERS
B. MASS MEASUREMENT	
The student should be able:	
1. to identify the basic units of mass measurement (mg, g, kg, t);	
2. to estimate various mass measurements;	Examples: 1) How heavy are you? 2) How heavy is your book?
3. to perform measurements of mass using scales;	
4. to convert from one basic mass unit to another.	Limit to mg, g, kg, and t.
C. CAPACITY	
The student should be able:	
1. to identify the basic units for measuring capacity (mL, L);	
2. to estimate the capacity of various containers;	Teachers are encouraged to have students first estimate and then measure the capacities of milk cartons, cups, drink bottles, spoons, etc.
3. to perform measurements of capacity;	
4. to convert from one capacity unit to another.	Limit to mL and L.

C.V. TRANSPORTATION: THE AUTOMOBILE (1½ modules)

The core unit on transportation centres around the automobile. The procedure for obtaining a driver's licence, the licence itself, the Manitoba merit-demerit system, the process of purchasing and insuring a vehicle, and the calculation of operation and maintenance costs are all covered in this practical and relevant unit.

OBJECTIVES	NOTES TO TEACHERS
A. DRIVER'S LICENCE The student should be able:	
1. to discuss the completion of an application form for a driver's licence;	Discussion could centre on the Driver Training Manual as well as on the process and cost of obtaining a driver's licence. A suggested resource person is the local Driver Education Instructor.
2. to interpret the information on a driver's licence;	Discuss classes of licences, restrictions, change of address/name, donor consent and renewal.
3. to discuss the Manitoba merit-demerit point system.	The following notes are provided courtesy of the Manitoba Motor Vehicle Branch.

MANITOBA'S MERIT POINT SYSTEM

Manitoba introduced a new "Merit Point System" effective September 1, 1978. The Merit Points are an incentive for drivers to maintain a clear driving record. A merit point will be awarded to a driver every two years. A driver may have a merit point awarded to his/her present driving record file if he/she meets the following criteria:

1. If a driver has no demerit points on his/her record and has not been assessed any demerit points due to accidents and/or convictions during a two-year period.
2. If a driver held a valid and continuous driver's licence of any class except Class 7 (instruction permit) during this two-year period.
3. If a driver was not suspended, prohibited from driving or had his/her licence cancelled during this two-year period.

4. If a driver did not have a surcharge on his/her licence, due to accidents during this two-year period.

To be awarded a merit point on September 1, 1978, a driver must meet the above criteria retroactive to September 1, 1976.

Additional merits will be awarded for each ensuing two-year period if a driver maintains the criteria outlined. A driver may earn a maximum of 5 merits. When a driver becomes eligible for a further merit, no additional merit will be awarded. He/she will be issued a certificate of merit to acknowledge his/her driving record.

Any merit points earned will be applied to any subsequent demerits that a driver may be assessed. The demerit total will be reduced at a rate of 2 demerits for each merit point. Any merit points which are applied toward demerit points will be removed from the driver's file.

Autopac - Additional Premiums of Demerit Points

Persons with bad driving records pay an additional premium for their driver's licences. The points assessed are based upon convictions for traffic violations. The additional premium charged on the driver's licence for demerit points is in accordance with the following table:

Demerit Points	Premium Assessment (1981) (in Dollars)
0 - 5	Nil
6	125.00
7	137.00
8	150.00
9	162.00
10	187.00
11	212.00
12	237.00
13	262.00
14	287.00
15	312.00
16	337.00
17	362.00
18	387.00
19	412.00
20	437.00
21 and over	465.00

By this system of assessing additional premiums, Autopac penalizes a person with a bad driving record and ensures that a good driver pays less.

MANITOBA'S POINT SYSTEM

Drivers who accumulate 6 points have their entire driving record reviewed and are normally called in. At this time they may be asked to take the complete driving examinations. Their records are reviewed with them, and they are counselled as to how they can improve their driving. They are then placed on probation which can vary from 12 to 36 months.

The number of points assessed against a driver is reduced after 12 months of accident- and conviction-free driving according to the following table:

Points	
1, 2 or 3	Reduced to 0
4	Reduced to 1
5	Reduced to 2
6	Reduced to 3
7 or 8	Reduced to 4
9	Reduced to 5
10	Reduced to 6
11 or 12	Reduced to 7
13	Reduced to 8
14	Reduced to 9
15 or 16	Reduced to 10
17	Reduced to 11
18 or 19	Reduced to 12
20	Reduced to 13
21 or 22	Reduced to 14
23	Reduced to 15
24 or 25	Reduced to 16
26 or 27	Reduced to 17
28, 29 or 30	Reduced to 18
31 or more	Reduced BY 13

To get updated figures contact your local Autopac agent or send for "Autopac-Your Automobile Insurance Coverage" which is a booklet available from: The Manitoba Public Insurance Corporation, Box 6300, 9th Floor, 330 Graham Avenue, Winnipeg, Manitoba R3C 4A4.

OBJECTIVES	NOTES TO TEACHERS
B. PURCHASE AND INSURANCE The student should be able:	
1. to identify sources of information for the purchase of an automobile;	Sources include newspaper want ads, car dealers, Consumer's Report, and friends, neighbors or relations.
2. to interpret vehicle sales ads;	Given a particular ad, the student should be able to transform the symbols and abbreviations into words. Also, students could write their own classified ads as a project.
3. to identify factors involved in the choice of a vehicle;	Factors such as purpose, size, fuel consumption, cost, and new vs. used should be discussed. Suggestions on what to do when examining a used car follow: . Always inspect a car in the daytime, outside in natural light. . Make most visual checks before starting the engine. . Be sure the engine is cold before starting it. . Check the condition of the tires for wear and for even wear at all corners of the car. . Look for leaks or puddles under the car. Check the exhaust system. . Wear clothes that can get dirty when inspecting a car. . Use everything that would normally be used (door, seat adjuster, windows, heater, radio, etc.). Check to make sure that the controls used daily work properly since their malfunction can be annoying. . Inspect the cold engine for leaks, bad hoses, frayed wiring, signs of overheating (rust-colored residue around the front of the engine and radiator) and signs of carburetor fires (bubbled paint on hood above carburetor). . Check the oil and automatic transmission dipsticks for low levels and smell them for burnt odors. . Check the battery and cables for wear and corrosion. . Start the car and observe any starting problems. . Always test the car with the radio off. . Have someone follow to see if the exhaust smokes and to see if the car tracks straight. . Drive the car under as many different conditions as possible to see how the engine, transmission, brakes and steering work. . Always assume that any problem is worse than it seems and that it will cost twice as much to fix as the seller claims. Lower the price of the car to allow for the proper repair of any recognizable problems. . Always take someone along to inspect a car for potential purchase. . Get a qualified mechanic or car clinic to check the car out.

OBJECTIVES	**NOTES TO TEACHERS**
4. to discuss sales agreements and consumer protection laws involved in the purchase of a vehicle;	Discuss both dealer sales agreements and private bills of sale. Resource: "Wheels and Deals" in videotape or 16 mm film. Available from: Consumer Affairs Division, 1023-405 Broadway Avenue, Winnipeg, Manitoba R3C 3L6.
5. to examine the financing involved in the purchase of a vehicle;	Discussion could centre around who provides the most economical loan. (Sources could include banks, credit unions, trust companies, dealer's financial organizations, finance companies and parents.) Local bank, credit union and automobile dealership representatives could be invited to serve as class speakers.
6. to discuss Autopac insurance coverage.	Discussion should include: preferred, all-purpose, farm and business coverage; optional coverage; third party liability insurance; procedures for dealing with accidents within and outside Manitoba; transfer of vehicle registration; change of address/name; assessment of a surcharge based on accident involvement; and situations where coverage is invalid. Teaching Suggestion: Assign specific topics to students, either individually or in groups, and have them report back to the class.
C. MAINTENANCE The student should be able:	
1. to distinguish between "fixed" and "variable" costs;	"Fixed" costs include: insurance, licence and registration, parking and storage, and depreciation. "Variable" costs include: gasoline and oil, regular maintenance, and repairs.
2. to calculate total "fixed" costs given the necessary information;	Calculations should include annual charges such as insurance and registration, and any parking or storage charges, etc.
3. to calculate the cost of a gasoline fill-up;	Example: 50.5L @ 36.3¢/L costs $18.33

OBJECTIVES	NOTES TO TEACHERS
4. to determine the fuel economy in L/100 km;	Example: Fuel economy in L/100 km = $\frac{\text{No. of Litres Used}}{\text{No. of km Driven}} \times 100$ Students should be aware that the lower the numbers, the better the fuel efficiency of the vehicle. Problem: Jane's Rabbit is rated at 5.6 L/100 km. John's Grand Prix is rated at 11.8 L/100 km. Using current rates, determine how much it costs each to drive 100 km. Which car costs less to drive and by how much?
5. to calculate the cost of specific repairs.	The cost of both labour and parts should be discussed so that students recognize the obvious savings that can be made by doing their own repairs.

C.VI. EMPLOYMENT (1 module)

This module deals with problems students will encounter when searching for a job and when on the job. Op.11 Employment can be used as a follow up to this module. Guidance personnel could be invited to assist the classroom teacher(s) by serving as class speakers or by providing copies of blank employment forms from their offices.

OBJECTIVES	NOTES TO TEACHERS
A. FORMS The students should be able:	
1. to complete a standard job application form;	At this stage, the conceptual framework should be kept simple. Teachers may want to discuss available resources for job finding, jobs appropriate for the individual, the reasons for social insurance numbers, etc. (See also Op.11 Employment.)
2. to discuss the completion of an application form for a social insurance number;	
3. to complete a current TD-1 form.	
B. HOURLY WAGES The student should be able:	
1. to calculate total wages over different periods of time (weekly, bi-weekly, semi-monthly, monthly) given a regular hourly rate;	To meet these objectives, students will have to be able to compute with decimals and/or fractions, to calculate minutes as parts of an hour, and to understand the relationship between regular, overtime, and special hourly pay rates.
2. to calculate the number of hours worked using a time card;	
3. to compute total wages with overtime.	

Objectives

C. PAY CHEQUE STUBS

The student should be able:

1. to identify gross pay;
2. to identify compulsory deductions (e.g., CPP, UIC, Income Tax);
3. to identify miscellaneous deductions (e.g., charitable donations, bonds, pension, union dues);
4. to calculate net pay given gross pay and deductions.

Notes to Teachers

The teaching of these objectives could involve observation of and calculation from a variety of sample pay cheque stubs. Only basic concepts of CPP, UIC, Income Tax and voluntary deductions are expected to be dealt with at this stage.

Reference:

Thomas F. Elrick, Forms in Your Life: A Student Workbook and Guide to Everyday Forms (Toronto: Globe/Modern Curriculum Press, 1981).

C.VII. MONEY MANAGEMENT: BANKING (1 module)

Money management is introduced here through a concrete examination of personal banking. The majority of time should be spent on the actual completion of various banking forms and records.

Note: Teachers are encouraged to use specimen bank/credit union/trust company forms. These could either be obtained in quantity from the appropriate sources, or copied by hand for duplication.

OBJECTIVES	NOTES TO TEACHERS
A. BANK ACCOUNTS	
The student should be able:	
1. to discuss reasons for using banking institutions;	
2. to identify the three main types of bank/trust/credit union accounts;	The three main types of accounts are: i) chequing, ii) regular savings, iii) true or premium savings. The differences between and uses of these types of accounts should be discussed.
3. to simulate opening a specific bank account.	Students should complete account application forms for either a chequing or a savings account. The importance of the signature to be used (specimen signature) in cheque writing should be stressed. Teachers may also point out that it is possible to use one of two signatures in a joint account and multiple signatures in a club or business account.
B. DEPOSIT AND WITHDRAWAL SLIPS	
The student should be able:	
1. to complete a deposit slip;	Students should complete several deposit slips using both cheque and cash deposits. The importance of keeping deposit receipts (in case of any bank error or disagreement) should also be pointed out.
2. to complete a withdrawal slip.	The cash withdrawal form should be discussed in reference to several specific examples. Students should be made aware that deposits and withdrawals can be made using computer banking.

OBJECTIVES	NOTES TO TEACHERS
C. CHEQUES	
The student should be able:	
1. to write a cheque properly;	In a "cashless" society, more and more cheques are used. Therefore, the proper procedure for writing a cheque must be covered in detail.
2. to distinguish between signing and endorsing a cheque;	Students should be encouraged to write "void" across the face of all simulated cheques.
3. to record the amount of each cheque in a journal or register;	It is important to stress that after each cheque is written a separate entry should be made in a journal or register.
4. to calculate the account balance after each cheque.	In conjunction with recording cheque amounts, teachers should have students calculate balances. Keeping accurate cheque records helps to avoid the embarrassment of being overdrawn and is especially useful in the examination of personal or family expenditures. Some time could be spent in discussing how to stop payment on a cheque, how to void a cheque, and what kind of penalties there are for overdrawing an account. Other bank services (provision of traveler's cheques, certified cheques, money orders, bank drafts, and safety deposit boxes) could be discussed.
D. BANK STATEMENTS	
The student should be able to read the information contained in an end-of-month statement and to answer questions pertaining to the given information.	Through the examination of specific monthly bank statements, students should be able to find: the beginning balance, the amounts of cheques enclosed, deposits during the month, the service charge, and the ending balance. Note: Reconciliation of the bank statement with the cheque book record is suggested only as an optional topic. Although a necessary function of a conscientious consumer, reconciliation involves multiple operations that might be too difficult for many students at this particular time.

OBJECTIVES	NOTES TO TEACHERS

References:

1. A Student Guide to Banking - Students' Portfolio

   This portfolio includes blank bank forms and a short history of banking. Classroom sets of the portfolio in English or French are free to teachers requesting them on school letterhead. (See address under #2 below.)

2. A Student Guide to Banking - Teacher's Instruction Booklet

   The Teacher's Instruction Booklet, a manual for A Student Guide to Banking, includes completed forms with detailed instructions. One copy of this booklet in English or French is free to the teacher requesting it on school letterhead.

   Remember to order both of these reference materials well in advance of the time they are required.

   Write: Canadian Imperial Bank of Commerce
   Head Office
   Commerce Court
   Toronto, Ontario
   M5L IG9

   OR

   Contact: Local branch of the Canadian Imperial Bank of Commerce.

3. Thomas F. Elrick, Forms in Your Life: A Student Workbook and Guide to Everyday Forms (Toronto: Globe/Modern Curriculum Press, 1981).

Op.I. CLOTHING (½ module)

A "life-skills" approach to introducing each term should be used. Catalogues, sales flyers, and newspaper advertisements could be used as sources for obtaining various clothing prices.

OBJECTIVES	NOTES TO TEACHERS
A. PURCHASES AND SALES TAX	
The student should be able:	
1. to calculate the price of a single article of clothing;	Students should be encouraged to estimate costs associated with clothing purchases. See C.1 Estimation. Example: If items are priced at 3 for $31.98 what is the cost of one of these items?
2. to calculate the price of several identical items of clothing given the price of one item;	Example: If one item costs $1.98, how much would 7 of them cost?
3. to calculate total purchase cost.	Calculating sales tax may be done by: 1) reading the appropriate amount from the sales tax chart, 2) multiplying by 5% or 0.05, or 3) multiplying by 10% and dividing by 2.
B. COMPARATIVE SHOPPING	
The student should be able:	
1. to distinguish between the regular price and the sale price and to calculate the saving;	Students should read various sale prices and recognize the different forms of advertising (e.g., a percentage off, stated sales price, dollars saved off regular price).
2. to calculate the sale price given the regular price and the rate of discount;	

OBJECTIVES	NOTES TO TEACHERS
3. to distinguish between prices for the same or similar clothing merchandise from one store to another.	Identical (or similar) articles of clothing may be priced differently from one store to another because some stores may be able to buy in larger quantities than others. Students should understand the importance of "shopping around." Project Suggestions: 1) Students could calculate what it would cost to make a specified article of clothing. In this way comparisons between manufactured and "self-made" products can be made. Students should realize that "self-made" items are not always cheaper. 2) Students could discuss several factors that should be considered when buying clothes to complete a wardrobe, such as climate, seasonal changes, quality, cost, and fashion. Reference: Saskatchewan Dept. of Manpower and Immigration, Lifeskills Coaching Manual (Prince Albert, Sask.: Queen's Printer, 1967). 600 pp; approx. $13.00.

Op.II. EMPLOYMENT (1 module)

Schools offering Work Education programs may have covered this topic preparatory to sending students out on Work Experience. Those that have will not find it necessary to cover this topic as part of their mathematics program.

OBJECTIVES	NOTES TO TEACHERS
A. APPLYING FOR A JOB The student should be able:	
1. to use the want ads to find a job;	This topic could be approached through individual student projects. Students could locate a particular job from the classified section and complete both a written letter of application and a resumé based on their own particular work experiences and training.
2. to complete a standard written letter of application;	
3. to complete a personal record (resumé);	Sample resumés and guides to acceptable writing styles and procedures can be obtained from Guidance, Language Arts or Business Education personnel. Information should include personal information, education, previous employment or record of work, and references.
4. to discuss factors involved in a job interview;	Job interviews could be simulated to help the students get a "feel" for the questioning process. Teachers with access to VTR equipment could tape these interview experiences, play them back before the class, and use them to initiate class discussion. Creative Job Search Technique - Teachers may wish to contact Canada Manpower for a presentation in "Creative Job Search Techniques." Apply: Canada Employment and Immigration Commission, 344 Edmonton Street, Winnipeg, Manitoba R3B 2L4.
5. to identify several employee benefits.	Examples: sick leave, compassionate leave, maternity and paternity leave, dental plan.

OBJECTIVES	NOTES TO TEACHERS
B. INCOME FROM EMPLOYMENT The student should be able:	
1. to identify major forms of job payment including: a. hourly wages, b. salary, c. commission, d. salary plus commission, e. piecework, f. self-employment (farmers, fishermen, trappers);	Since hourly wage calculations have already been dealt with intensively in the first employment module, additional work may not be needed here. Hourly wages are only mentioned here as a method of payment.
2. to calculate weekly or monthly earnings given a specified salary (contract) for a set time period;	For example, given an annual salary, the student should be able to calculate the monthly and the weekly wage.
3. to calculate earnings given a flat rate of commission;	Note: It is anticipated that students will have to do some preliminary work with percent before beginning to calculate commissions. First, they should know how to change a given percent rate into either a decimal fraction or a common fraction. Only simple percent rates should be used. Example: $12\% = 0.12 = \frac{12}{100}$. Secondly, students should be able to calculate the amount of commission (C) given total sales (S) and the rate of commission (R). The formula $C = S \times R$ could be used although it is up to the teacher whether or not to state it as such. Calculators should be used.

OBJECTIVES

4. to calculate earnings given a basic salary plus a commission;
5. to calculate earnings from piecework.

NOTES TO TEACHERS

Example: Find the amount of commission sales on $150 if the rate of commision is 6%.

Method 1

$C = \$150 \times 6\%$
$= \$9.00$

Method 2

$C = \$150 \times 0.06$
$= \$9.00$

Method 3

$C = \$150 \times \frac{6}{100}$
$= \$150 \times \frac{3}{50}$
$= \$9.00$

Enrichment material could include working out graduated commissions; e.g., 6% of the first $1 000 of sales, 10% on the next $500 sales, 15% on sales in excess of $1 500.

References:

Durlynn Anema, Don't Get Fired (Janus Book Publishers). Available from SRA.

Kibrell and Vineyard, Succeeding in the World of Work (Bloomington, Ill.: McKnight and McKnight Publishing Co., 1970).

Saskatchewan Dept. of Manpower and Immigration, Lifeskills Coaching Manual (Prince Albert, Sask.: Queen's Printer, 1967). 600 pp; approx. $13.00.

Op.III. LEISURE ACTIVITIES (½ module)

Leisure activities provide many opportunities to develop mathematical concepts and skills since most involve scoring, comparative costs, measurements, ratings and averages.

This unit should be done occasionally throughout the term.

Note: Teachers may elect to integrate material from Op.III Leisure Activities with Op.VI Recreational Mathematics.

OBJECTIVES

The student should be able to interpret and to complete mathematical exercises associated with a number of leisure activities.

NOTES TO TEACHERS

Students could prepare classroom presentations involving the following:

- the completion of charts and graphs;
- the preparation of comparative cost studies;
- the discussion of hidden costs/benefits involving money, time or energy;
- the interpretation of data;
- the examination of advertising materials.

Teachers are encouraged to draw on student interest and knowledge in discussions involving leisure activities. In addition, students should be introduced to activities that are unfamiliar to them so that their interest and knowledge will be expanded.

The comparative costs/benefits of leisure activities from each of the categories below should be examined:

1) Participation activities: (to be done at appropriate times)

basketball, billiards, boating, bowling, camping, curling, dancing, dragsters, fastball, fishing, football, golf, hockey, ice skating, pinball, racquetball, roller skating, scuba diving, skiing, snowmobiling, soccer, swimming or tennis.

2) Hobby/creative/social activities:

bingo, bridge, ceramics, checkers, chess, club membership, collections of butterflies, coins, spoons or stamps, craft work, decorating, music lessons, painting, photography, pottery, purchase of records or tapes, restoration of cars, sewing, taking short courses or travel.

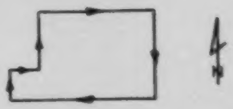
3) Spectator activities:

ballet, concerts, dinner shows, horse racing, movies, professional sports, theatre or watching T.V.

OP.IV. APPLICATIONS OF METRIC MEASUREMENT (1 module)

Estimation is an important feature of measurement. Refer to C.1 Estimation.

Care should be taken to use suitable measurement units at all times.

OBJECTIVES	NOTES TO TEACHERS
A. TEMPERATURE The student should know appropriate Celsius temperatures for a variety of situations.	Examples of situations associated with a definite temperature or temperature range include: - a hot day in July - a day in January - the air inside a freezer - the normal temperature of a healthy person - a normal room temperature - the freezing, melting, and boiling points of water.
B. PERIMETER The student should be able: 1. to demonstrate an understanding of the concept of perimeter; 2. to calculate the perimeter of a rectangle, square, and irregular shape given the appropriate dimensions; 3. to solve problems involving perimeter.	Examples: 1) Find the distance travelled if you go 2km N, 4 km E, 3 km S, 5 km W, 1 km N and 1 km E. 

OBJECTIVES

C. AREA

The student should be able:

1. to demonstrate an understanding of the concept of area;
2. to calculate the area of a rectangle given the length and width;
3. to solve problems using the area of a rectangle.

NOTES TO TEACHERS

2) Find the length of fencing needed for a rectangular dog run 10 m long and 8 m wide.

3) A farmer has 800 m of fencing and wishes to make a rectangular horse corral. What different dimensions could be made with this length of fencing?

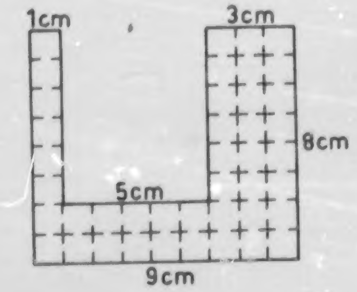
Teachers may need to provide concrete materials to develop the concept of area (covering a flat surface with unit squares). Tiles, tessellations, polyominoes or tangrams may be used.

Examples:

1) Calculate the area of sod needed to make a lawn 8 metres wide and 10 metres long.

2) Calculate the amount of carpet needed for a house.

3) What is the area of this metal plate?



Additional problems can be found in vocational mathematics textbooks.

OBJECTIVES

D. CAPACITY AND VOLUME

The student should be able:

1. to demonstrate an understanding of the concept of volume;
2. to calculate the internal volume of a rectangular solid;
3. to solve problems involving the volumes of rectangular solids.

NOTES TO TEACHERS

Concrete materials may be needed to develop the concept of volume (cubes could be placed into rectangular boxes to give the concept of space filling).

Example: How many 1 cm cubes would be needed to fill a box 10 cm by 8 cm by 5 cm?

Examples:

1) a. A carton containing a transistor radio is 10 cm by 20 cm by 8 cm. Calculate the volume of the carton.
   b. Cartons containing transistor radios are to be packed for transportation in larger boxes, 50 cm by 80 cm by 40 cm. How many of these cartons can be placed in the larger box?

2) How many cubic metres of concrete would be needed to make the floor of a garage 6 m long and 4 m wide if the concrete is to be 15 cm thick?

Op.V. MONEY MANAGEMENT: BUDGETING (1 module)

OBJECTIVES | NOTES TO TEACHERS

A. PERSONAL BUDGET

The student should be able:

1. to make a list of income and expenditures;

It is suggested that students be asked to prepare a list of their income and expenditures a week or two in advance so that they have a basis for discussion. Teachers will need to provide data for students without income.

Example:

Date	Details	Income	Expenditures
Oct. 10	Balance on hand	20.25	
10	Lunch		.95
11	Record		7.35
11	Lunch		.80
12	T-Shirt		6.00
12	Baby-sitting	7.00	
14	Lunch		1.05
16	Movies, etc.		5.00
	TOTALS	27.25	21.15

Students should recognize categories into which expenditures fall: food, clothing, entertainment, transportation.

2. to complete a budget showing estimated income and expenditures;

Have students complete a budget showing estimates of income and expenditures for the next month.

Example:

Elizabeth estimates that her income for November will be: allowance, $20.00; baby-sitting, $40.00; part-time job, $50.00. She thinks her expenditures might be: lunches, $15.00; record, $7.50; bus fares, $10.00; movies (twice) $7.00; cosmetics, $5.50; T-Shirt, $5.00; birthday gift for boyfriend $20.00; savings towards holiday, $30.00.

OBJECTIVES

3. to calculate the balance in a personal budget.

B. FAMILY BUDGETS

The student should be able:

1. to make a list of regular expenses;

NOTES TO TEACHERS

Complete a budget placing expenditures into the following categories: food, clothing, entertainment, transportation, savings, gifts.

BUDGET FOR NOVEMBER

Estimated Income		Estimated Expenditures	
Allowance Baby-sitting Part-time job	20.00 40.00 50.00	Gifts Food Clothing, etc. Entertainment Transportation Savings	20.00 15.00 10.50 14.50 10.00 30.00
TOTALS	110.00		100.00

Categories chosen will vary according to the interest of the students. The important idea to stress is looking ahead in order to plan finances.

"Family" includes one-person families. Discussion of larger families could be included if students and teacher so wish.

Discuss the categories into which monthly expenditures may be classified: shelter, food, clothing, household utilities, transportation, gifts and savings for annual expenses such as: Autopac, insurance on belongings, and savings for holidays and/or emergencies.

Students should be encouraged to use the newspaper to determine appropriate shelter costs and wages.

OBJECTIVES	NOTES TO TEACHERS
2. to complete a budget showing regular and discretionary expenditures.	Students could estimate the monthly expenditures and complete budgets for: 1. a person paying board and room who earns $125 per week; 2. a student with a part-time job bringing in $450 per month who shares an apartment with two others; 3. a single person earning $12 000 per year, who lives in an apartment costing $250 per month that includes hydro, and has a loan for which he or she pays $100 per month.

Op.VI. RECREATIONAL MATHEMATICS (½ module)

The primary aim of this unit is to use recreational mathematics activities and problem solving exercises to encourage logical thinking in students. Activities should be chosen that will heighten student motivation. Students should be actively involved. This unit should be done occasionally throughout the term.

OBJECTIVES	NOTES TO TEACHERS
The student should be given the opportunity:	
1. to solve mathematical puzzles;	Student interest and ability will dictate the level of difficulty of the puzzles used. Examples could include code-breaking problems, color problems, commercial puzzles such as Soma, Instant Insanity and Rubik's Cube, logic puzzles, magic squares, match-stick problems, number puzzles and string/wire puzzles.
2. to play mathematical games;	Examples could include: Backgammon Calculator games such as the following: 1) Estimation Game - 2 players. Two players each have 30 seconds to estimate a product or quotient of 2-, 3-, or 4-digit numbers. The player with the closer estimate wins a point. The first player with 10 points wins. 2) Target Game. Players alternate pressing a number and ⊗ to reach a target number. The first player to come within 1 of the target number wins. 3) Game of 21 - 2 players. On each turn a player can push only one of the three keys 1, 2 or 3 along with the ⊕ key. The first player to reach 21 is the winner. Have students determine the winning strategy. 4) Game of 50 - 2 players. Same as above except using the keys 1, 2, 3, 4, 5 and 6. Mastermind

OBJECTIVES	NOTES TO TEACHERS
	Aware *Othello* *Cribbage* *Monopoly*
3. to participate in geometric activities.	Students could attempt to solve construction puzzles, dissection problems, packing problems and spatial relation exercises; they could experiment with symmetry activities, polyominoes, tangrams and tessellations, mazes, string-wire art, 3-D models and paper folding. Books containing recreational math puzzles may be obtained from local bookstores and libraries. The following are recommended: *Games*, Box 10147, Des Moines, Iowa, U.S.A., 50349 (A two year subscription for 12 issues was $14.97 in 1982). Peter Van Delft and Jack Botermans, *Creative Puzzles of the World* (New York: Harry N. Abrams, 1978).

Op.VII. GATHERING AND RECORDING DATA (½ module)

Teachers could initiate discussion about the census and public opinion polls; reasons for collecting data and bias in collection procedures could also be discussed.

OBJECTIVE	NOTES TO TEACHERS
A. GATHERING DATA	
The student should be able:	
1. to list several kinds of data that are commonly gathered;	
2. to distinguish between factual data and opinion;	
3. to name several agencies that collect data and to state why they need data;	Federal, provincial, or municipal governments; manufacturers trying to predict markets; political parties trying to measure voter support; Neilson ratings.
4. to list several methods of collecting data;	Interview, questionnaire, telephone polls, census forms, opinion polls, etc.
5. to collect data by counting and/or questionnaire.	
B. RECORDING DATA	
The student should be able:	
1. to organize data into tables with the headings: description, tally, and frequency;	This section could be dealt with as a group project with the students pooling their data to create a larger set.
2. to draw bar graphs and line graphs.	It may be necessary for teachers to give the appropriate scales for the bar or line graphs.

Op.VIII. TRANSPORTATION (1 module)

OBJECTIVES	NOTES TO TEACHERS
A. THE AUTOMOBILE - NEW/USED/LEASED The student should be able: 1. to identify reasons why people buy new/used/leased vehicles;	1) Reasons for buying a new car include: - the manufacturer covers repairs during the warranty period; - a financing agreement can be arranged with the car dealer; - a new vehicle is more fuel efficient; - an owner feels proud of a new vehicle and avoids buying someone else's problem. 2) Reasons for buying a used car include: - lower price, and/or monthly payments; - most depreciation has been absorbed by the previous owner. 3) Reasons for leasing a car include: - no down payments, and no interest charges; - the leasor covers the major repairs; - a large portion of the lease payments can be applied later to the purchase of the car; - leased vehicle is usually a new car. Consideration can be given to the advantages and disadvantages of the three choices above.
2. to compare cost over a 1, 3 or 5 year period for new/used/leased vehicles.	Let vehicle costs include purchase price, financing, and anticipated repairs. Have students calculate the total cost of a new vehicle; have them use car ads, dealer's black book, etc. to estimate the value of the car after each time period. The same procedure can be followed for used cars. Leased vehicle costs include the total of monthly lease payments and repairs not covered by the leasor. Remind students that the driver has no equity in the car after each time period.

OBJECTIVES	NOTES TO TEACHERS
B. PLANNING A TRIP USING PUBLIC TRANSPORTATION The student should be able:	
1. to interpret bus schedules;	Both urban and rural schools should use bus schedules appropriate to their locations to illustrate departure and arrival times.
2. to interpret train or airline schedules;	Differences in cost according to the type of fare (first class, economy class, day coach, etc.) could be discussed. Students could calculate and then compare total costs for several different situations.
3. to plan a route using appropriate bus/train/airline schedules.	Students should consider the number of transfers, waiting time, and the scheduling involved in these modes of transportation. Examples: 1) a trip from Winnipeg to Lethbridge involves: - choosing between air and train travel from Winnipeg to Calgary - consulting Greyhound schedules for appropriate departure time from Calgary to Lethbridge 2) a bus trip within an urban centre often involves using 2 or more routes. Discuss transfer points.
C. PLANNING A TRIP BY AUTO/TRAIN/AIRPLANE/BUS The student should be able:	
1. to interpret a highway map;	For a given route, students should be able to locate the major highways and junctions.

OBJECTIVES

2. to calculate distances between given cities on a map;

3. to approximate travel time for given distances;

4. to approximate total travel costs for a particular trip.

NOTES TO TEACHERS

Calculating distance on a map may be done in three ways:

1) adding given individual distances to get accurate total distances.

2) measuring the straight distance (using ruler) between the two cities and then calculating the distance by using the scale. This gives an approximate figure.

3) reading the distance between two centres if given on a distance chart.

Given the rate and the distance, the student should calculate the approximate time for the journey. Refer to C.I Estimation.

Note: This may be done as a class project, the length and complexity of treatment depending on the students' interests and abilities.

Have students compare 2 methods of travel as in the following example:

Plan a trip from Winnipeg to Toronto. First, plan the trip by automobile and include motel and/or camping costs, gasoline, food, etc. Next, plan the trip by VIA rail.

Note: Allow students to make any assumptions they wish concerning the details of this trip; for example, they may wish to plan a two-week trip which involves staying 4 days with relatives in Toronto.

Have students complete an itinerary and expense sheet.

Sources of information include:

- Manitoba Motor League (MML) maps
- American Automobile Association (AAA) books
- a travel agent might agree to offer suggestions and information.

mathematics 201

MATHEMATICS 101

THE 01 MATHEMATICS PROGRAM

01 Mathematics is an amalgamation of previous 01 (general), 02 (business), and 03 (vocational) mathematics programs in Manitoba. It is a truly general mathematics program, incorporating the best from several areas of mathematics and leading to a range of post-secondary education programs, vocational training and employment, as well as to more general life skills. This program is needed to familiarize students with a wider variety of mathematics topics and, thereby, to expand their future options.

Core topics of Mathematics 101 include arithmetic, consumer mathematics, geometry and algebra. To produce a practical program, attempts have been made to include practical applications to current problems from business, consumer, trade and vocational areas. In this way, all 01 students, not only those registered in business and vocational courses, have an opportunity to receive instruction in business and/or vocational mathematics topics.

The amalgamated 01 program is particularly beneficial to smaller schools which, because restricted to offering two courses in mathematics at any one grade level, have been in the past unable to offer business and vocational mathematics. It should be noted here, however, that larger schools may want to identify separate classes of 01 Mathematics with options dedicated to business, vocational or general mathematics.

GENERAL GOALS OF THE 01 PROGRAM

The student goals upon which this guide is based are included below. The program is designed:

1. to develop an awareness of the need/usefulness of mathematics;
2. to increase understanding of the concepts and to develop mastery of basic arithmetic skills using whole numbers, integers and decimal fractions through the solution of practical problems in the consumer, business, trade and vocational areas;
3. to increase proficiency in mental arithmetic and rapid calculation exercises;
4. to increase comprehension in the problem-solving process;
5. to increase proficiency in making reasonable estimations in both geometric and numerical activities;
6. to develop further understanding and facility with metric measurement and language;
7. to increase consumer awareness with reference to both the market and work places;
8. to develop an understanding and a mastery of basic algebraic skills;
9. to increase inductive and deductive reasoning and other thinking skills; and
10. to cultivate a positive attitude towards mathematics.

STRUCTURE OF THE MATHEMATICS 101 COURSE

Each module is designed for 14 periods (40 minutes each) of new work in a non-semestered system. This should allow approximately 4 periods per module for review, testing, diagnosis and enrichment. Ten modules comprise a full credit. Core modules are compulsory, and Mathematics 101 has the equivalent of 5 core modules. The remaining equivalent of 5 option modules are to be selected from the business, vocational and/or general mathematics topics.

The following chart gives a listing of the topics identified for the Mathematics 101 course and an indication of the allotted time in terms of modules.

CORE TOPICS (Complete each of the following.)

			Page References
C.I.	Arithmetic	(½ module)	96
C.II.	Consumer Mathematics	(1½ modules)	101
C.III.	Geometry	(1½ modules)	123
C.IV.	Algebra	(1½ modules)	133

OPTION TOPICS (Select the equivalent of 5 modules from the following.)

Calculator Mathematics			
Op.I.	Operations of and Applications for Calculators	(½ module)	145
Business Mathematics			
Op.II.	Basic Skills in Business Mathematics	(1 module)	147
Op.III.	Discount Calculations	(1 module)	153
Op.IV.	Merchandise and Service Business Calculations	(1 module)	163
Vocational Mathematics			
Op.V.	Measurement and Estimation	(1 module)	173
Op.VI.	Ratio and Proportion	(1 module)	187
Op.VII.	Percent Calculations	(1 module)	203
General Mathematics			
Op.VIII.	Graphing	(1 module)	209
Op.IX.	Estimation	(½ module)	222
Op.X.	Mathematics Experiments	(½ module)	226
Op.XI.	Spatial Relations	(½ module)	234
Op.XII.	Statistics	(½ module)	247

FEATURES OF THE 101 COURSE

1. Mathematics 101 approaches mathematics through practical problems and applications from the consumer, business, trade and vocational areas.

2. The order to be followed is to be determined by the student/teacher. Some classes may prefer to complete the core material first while others may wish to create greater variety and interest by integrating core and option topics. Units are purposely independent of one another in order to create a flexible mathematics program.

3. Even though topics are identified as ranging from ½ module to 1½ modules in length, they may be taught in a block or spread throughout the program.

4. Students are expected to achieve a reasonable mastery of the stated objective(s) for any module(s). The methodology, suggestions, and examples are included in an attempt to clarify the objectives and expectations.

5. Recommended time limits help to identify the extent (depth) to which objectives should be studied. All period references are to 40-minute class periods.

6. Option topics are not sequential. They may be taught in any order. For example, in Business Mathematics, Option III may be taught before or independently of Option II.

C.I. ARITHMETIC (½ module)

Teachers should realize that drill and practice play an important role in the mastery of computational skills, but that total reliance on drill and practice is not an effective approach to learning. The content identified in this section of the Mathematics 101 outline is review. Suggestions for teaching this material are listed below.

1. Use the material in this section to encourage student proficiency in problem solving with calculators. Refer to Op. I Operation of and Applications for Calculators. Students using calculators are essentially equal in terms of computational ability, speed, and accuracy. Calculator usage is encouraged throughout 01 Mathematics courses.

2. Integrate a review of the material into various sections of the course. In other words, review the outlined objectives just prior to their actual use in the course.

3. Devote a complete block of time (½ module) to reviewing the material. This review could be used to refresh student arithmetic capabilities or to diagnose student strengths and weaknesses in arithmetic computation.

Teachers should encourage students to recognize when developing an algorithm is more efficient than using a calculator. While endless drills in computation should be avoided, mathematical skills and concepts underlying computation algorithms should be stressed. Students who understand the process of computation and develop the skills of estimation are less likely to become completely dependent on the calculator and are well prepared for future learning.

In summary, attempts should be made to disguise practice in basic arithmetic operations by employing current consumer applications to develop mathematical awareness, to stress mental arithmetic, to emphasize mathematical reasoning throughout computation, to place equal importance on estimation and correct answers, and to encourage confidence and proficiency in computing when using the calculator.

OBJECTIVES	NOTES TO TEACHERS
A. WHOLE NUMBERS The student should be able to demonstrate proficiency in the fundamental arithmetic operations of addition, subtraction, multiplication and division with whole numbers.	Exercises involving these concepts should be of a problem-solving nature and have practical application. Some exercises will need to have numbers rounded to whole numbers in order to fit in with this objective. Examples could include: 1) Performing mathematical calculations associated with sports or other leisure activities. Students could complete a basketball summary sheet, score 5-pin or 10-pin bowling games, correlate Yahtzee score cards, calculate ratios for bicycle gearing, etc. 2) Finding the cost of automobile fuel.

OBJECTIVES	NOTES TO TEACHERS

3) Tabling inflation costs. Students could complete the following table, considering inflation to be 10% over the previous year.

Gross Income		$15 000	
Income Tax Deduction	$2357		
Other Deductions	243		
Total Deductions	____	____	(subtract)
Net Income		____	
Inflation (10%)		____	(subtract)
Relative Income		____	

B. INTEGERS

The student should be able to demonstrate proficiency in the addition and subtraction of integers.

Teachers should stress order when comparing integers. Temperature scales and stock market quotations could be used to enhance the learning.

Students should be able to use the inequality relations correctly (i.e., $-8<0$ and $-4>-6$) and to arrange a series of integers in order.

C. DECIMAL FRACTIONS

The student should be able to demonstrate proficiency in the fundamental operations of addition, subtraction, multiplication and division with positive decimal fractions.

Teachers should emphasize differences in determining decimal placement among the operations of addition, subtraction, multiplication, and division.

Students should realize that division by a positive decimal fraction less than 1 will yield a number larger than the dividend.

Example: $\frac{3}{0.5} = 6.$

Commercial invoices, deposit slips, and passbooks containing both deposits and withdrawals could be used to lend reality to this topic.

D. FRACTIONS

The student should be able to convert fractions to decimal fractions.

Whenever repeating decimal fractions occur, students should employ approximate notation, or use repeating decimal notation, or write answers correct to, say, two or three decimal places.

Teachers could use forms and practical problems involving time cards, scale drawing conversions, stock market quotations, etc., to review this material.

OBJECTIVES | **NOTES TO TEACHERS**

Example:

Jane buys $3\frac{1}{2}$ metres of cotton polyester and $1\frac{2}{3}$ metres of velour to make a winter wardrobe. Find the total number of metres she buys. (Convert fractions to decimal fractions correct to 3 decimal places in the solution of this problem.)

Solution:

$$\begin{array}{r} 3\frac{1}{2} \\ +\ 1\frac{2}{3} \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 3.5 \\ +\ 1.667 \\ \hline 5.167 \end{array}$$

Jane buys approximately 5.157 m of cloth.

E. MULTIPLICATION AND DIVISION OF SIGNED NUMBERS

The student should be able to multiply and to divide signed numbers (both whole numbers and decimal fractions).

Students are expected to know the rules for multiplication and division of signed numbers. Mastery is expected for integers and decimal fractions.

Examples: Multiply or divide as indicated:

$(-2)\times(3)$; $(6)\times(-8)$; $(-15)\times(-2)$; $(-3)\times(-7)\times(+4)$;

$(4.2)\times(-3)$; $(-8)\times(-0.3)$;

$(27)\div(-3)$; $(-3.6)\div(-2)$; $(21)\times(5)\div(-0.7)$.

Note: Students must realize that:

1) Parts of a fraction with no expressed sign are assumed to be positive.

2) Any fraction has 3 signs associated with it: one in the numerator, one in the denominator and one overall.

Example: $\frac{-a}{b}$ means $+\frac{-a}{+b}$.

3) In a negative fraction, the minus sign can be placed in the numerator, denominator or in front.

Example: $\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$.

OBJECTIVES

F. ORDER OF OPERATIONS

The student should be able to evaluate numeric expressions involving several operations.

NOTES TO TEACHERS

After reminding students of the order of operations, teachers should give them a selected set of exercises that review operations on whole numbers, integers and decimal fractions.

Examples:

$$12 + 7 - 3(2) = 13$$

$$8 + 9(2) + 4 = 30$$

$$8(5) - 6(3) = 22$$

$$30 - 4(8-2) = 6$$

$$3(9+16-5) = 60$$

$$12 + (-8-3)7 - 20 = -85$$

$$0.2(6+3) - (8-9) = 2.8$$

$$\frac{16-4}{3(2)} + 8(0.2) = 3.6$$

$$\frac{10-4}{17-15} + \frac{1.8+42}{0.6(2)} = 39.5$$

$$\frac{4(7+2)}{3+9} - \frac{(19-5)6}{4+2} = -11$$

OBJECTIVES

G. PERCENT

The student should be able:

1. to identify percent as the numerator of a fraction with 100 as the denominator;
2. to convert percents to decimal fractions and vice-versa;

NOTES TO TEACHERS

Teaching suggestions for percent are to be found in Op.VII Percent Calculations. Objectives outlined here could be integrated with those in Op.VII.

OBJECTIVES	NOTES TO TEACHERS
3. to determine the value of a stated percent of a given number.	"Percent" without applications is of little value to students. Practical applications of percent calculations are integrated throughout this guide. Teachers' experience, available textbooks and support resources will determine the method used in solving percent problems. Terminology related to percent rate, base, and percentage (amount) is used in this guide as follows: Percent Rate x Base = Percentage (Amount) 20% of 35 = 7
H. ESTIMATION The student should be able:	
1. to round off given numbers to stated place values;	Teaching suggestions for estimation are to be found in Op.IX Estimation.
2. to round off numbers and then to perform the appropriate arithmetic operations.	One purpose of estimation is to provide a quick check on the reasonableness of answers in computation and problem solving. Estimation is a mental process, not a paper and pencil procedure. It is a difficult process for most students in that they lack prerequisite skills, including the ability to round off numbers, proficiency in operating with multiples of 10, 100, 1000, etc., and facility with mental arithmetic. The responsibility for developing estimation skills must be shared among all mathematics teachers. Teachers must continually stress the use of rounding off and estimation in arithmetic computation and problem solving.

C.II. CONSUMER MATHEMATICS (1½ modules)

This section of the 101 course is designed to provide students with opportunities to solve consumer problems. A knowledge of consumer mathematics should help students to become intelligent consumers. The unit concentrates on income, methods of budgeting, consumer purchasing, and banking.

OBJECTIVES	NOTES TO TEACHERS
A. INCOME The student should be able: 1. to calculate gross wages earned, given:	
a) hours worked and rate per hour,	Example: Sally Brown worked 30 regular hours in one week at $4.80 an hour. What are her gross earnings for the week?
b) hours worked, including overtime, and the rate per hour for weekly and daily time periods,	Examples: 1) Frank Hall worked 36 hours during a regular 5-day week and 4 hours on Saturday. Frank's regular pay is $5.60 an hour. If the company for which Frank works pays time and a half for all Saturday work, what is his weekly gross income? 2) Denise Roget recently worked a 40-hour week (8 hours each on Monday through Friday). Her regular pay is $4.50 an hour. During the same week she worked 6 hours on Sunday for which she was paid double time. What are her gross wages for the week? 3) Jim Johnson worked 8 hours on Monday; 9 on Tuesday; 10 on Wednesday; 6 on Thursday; 9 on Friday, and 5 on Saturday. His regular pay is $6.10 an hour. If the company for which Jim works pays time and a half for any hours worked over 37½ hours per week, what are Jim's gross wages for the week?

OBJECTIVES

c) rules for late and early arrival, time cards, and rates of pay;

NOTES TO TEACHERS

The expression of times may be given using:

1) customary units involving a.m. and p.m.
8:00 a.m. to 5:00 p.m. may be abbreviated 8:00 to 5:00 in some textbooks.

2) 24-hour clock
06 30 represents 6:30 a.m.
18 30 represents 6:30 p.m.
Note: Some authors write 18 30 as 18:30.

Teachers are reminded that going metric does not imply that we necessarily switch to the 24-hour clock and that most businesses still express time conventionally on their time cards. Examples should employ customary and 24-hour notations.

The following table outlines sample procedures used for late arrival/early departure of the employee(s).

Minutes Late	Penalty
1 to 7 minutes	no deduction
8 to 22 minutes	¼ hour deduction
23 to 37 minutes	½ hour deduction
38 to 52 minutes	3/4 hour deduction
53 to 60 minutes	1 hour deduction

The same principles apply to early departures. For example, if an employee leaves at 3:46 instead of 4:00, one-quarter hour is deducted from the total hours worked for that day.

OBJECTIVES

NOTES TO TEACHERS

Example:

Find the total hours worked from the following time card(s) for Dorothy Miller for the weeks shown and compute her gross weekly earning if her regular workday is from 8:00 to 5:00 and overtime is at time and a half, using the above sample of pay procedure.

Employee No. 766-05-7604

Name Dorothy Miller

Week Ending March 14, 19__

Days	Regular				Overtime		Hours
	In	Out	In	Out	In	Out	
Mon.	8:15	12:00	1:05	5:00			
Tues.	8:32	12:01	1:01	5:02			
Wed.	8:12	12:06	1:08	4:00			
Thurs.	8:26	12:02	12:50	4:30			
Fri.	8:17	12:01	12:45	4:15			
Sat.					1:00	6:00	
Sun.							

	Hours	Rate	Earnings
Regular		3.75	
Overtime			
Total Hours		Gross Earnings	

OBJECTIVES

NOTES TO TEACHERS

Following is an example of the same time card using the 24-hour clock notation to indicate time.

Employee No. 766-05-7604

Name Dorothy Miller

Week Ending March 14, 19__

Days	Regular				Overtime		Hours
	In	Out	In	Out	In	Out	
Mon.	08:15	12:00	13:05	17:00			
Tues.	08:32	12:01	13:01	17:02			
Wed.	08:12	12:06	13:08	16:00			
Thurs.	08:26	12:02	12:50	16:30			
Fri.	08:17	12:01	12:45	16:15			
Sat.					13:00	18:00	
Sun.							

	Hours	Rate	Earnings
Regular		3.75	
Overtime			
Total Hours		Gross Earnings	

Note: Students should be made aware of new trends concerning time cards or sheets. Further, many businesses do not deduct wages if an employee is late, etc. Instead the employee may be required to make up for time missed.

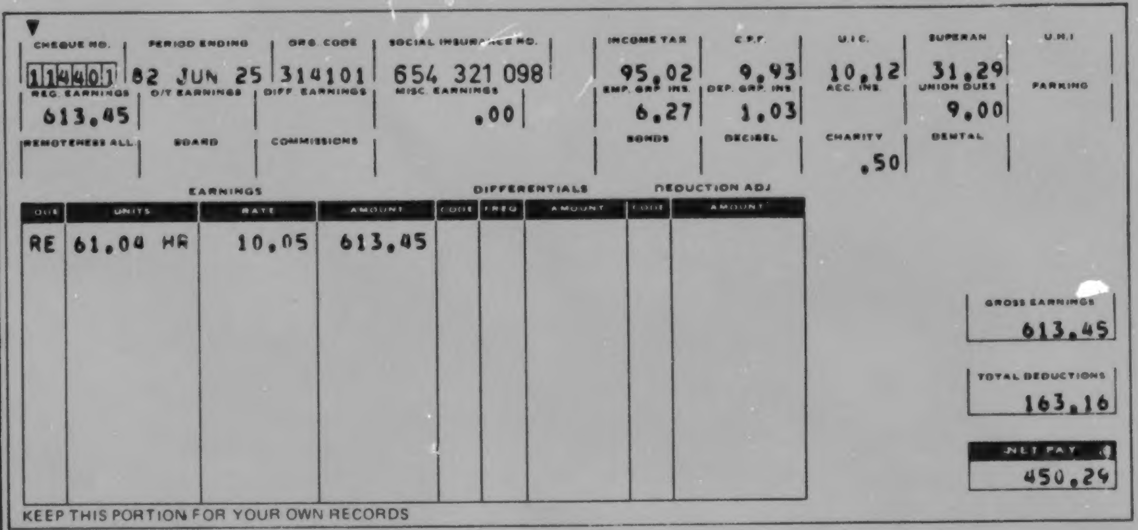
OBJECTIVES

2. to calculate net pay given gross pay and deductions;

NOTES TO TEACHERS

Subtraction is the only operation involved here. No tables for CPP or UIC should be used. Discussion could revolve around types of deductions including compulsory and non-compulsory ones. The idea is to leave students with the understanding that gross pay and take-home pay are not the same.

Following is an example of a payroll deduction sheet:



CHEQUE NO. 114401	PERIOD ENDING 82 JUN 25	ORG. CODE 314101	SOCIAL INSURANCE NO. 654 321 098	INCOME TAX 95.02	C.P.P. 9.93	U.I.C. 10.12	SUPERAN 31.29	U.H.I
REG. EARNINGS 613.45	O/T EARNINGS	DIFF. EARNINGS	MISC. EARNINGS .00	EMP. GRP. INS. 6.27	DEP. GRP. INS. 1.03	ACC. INS.	UNION DUES 9.00	PARKING
REMOTENESS ALL.	BOARD	COMMISSIONS		BONDS	DECIBEL	CHARITY .50	DENTAL	

EARNINGS				DIFFERENTIALS			DEDUCTION ADJ	
DUE	UNITS	RATE	AMOUNT	CODE	FREQ	AMOUNT	CODE	AMOUNT
RE	61.04 HR	10.05	613.45					

GROSS EARNINGS	613.45
TOTAL DEDUCTIONS	163.16
NET PAY	450.29

KEEP THIS PORTION FOR YOUR OWN RECORDS

OBJECTIVES

NOTES TO TEACHERS

3. to calculate new salary, given present salary and percent rate of increase:

Examples:

1) James Martin's salary for his first position after graduation was $8000 a year. After one year his employer granted him a 20 percent increase in his salary. Calculate his new salary. (Students should realize that the easiest way to calculate the new salary in this case is to multiply 8000 x 1.20.)

2) A firm increases the daily wages of its employees by 12½%. Find the new daily wages of a person who was earning $56.25 per day.

3) A wholesaler employed William Green as a salesman, agreeing to pay him $1200 a month and to increase his salary by 10% every 6 months. How much will William Green's monthly salary be at the end of one year? three years?

4. to calculate the percent rate of increase, given the amount of increase and the present salary.

Examples:

1) Presently, Robin Allen earns $15.00 an hour. According to the terms of a new salary agreement Robin will receive an increase of $3.00 an hour at the beginning of next month. What percent rate of hourly increase will Robin receive under the new salary agreement?

2) Mary Alden's present income is $200.00 a week. Mary received a notice from the payroll office that she would receive an increase of $25.00 a week. Find the percent rate of increase in her weekly income.

3) Jean Brown worked as a clerk-stenographer following graduation from high school at $900.00 a month. After two years, her earnings were $1200 a month. What was the percent rate of increase in her salary over the two-year period?

4) Garry Smith earns $200 000 a year as a hockey player. Next year his salary will be $250 000. Calculate the percent rate of increase in Garry's yearly salary.

For enrichment, teachers may wish to extend this section by including the following types of questions:

1) What is a clerk's original monthly salary and his/her present monthly salary if he/she receives an increase of $200.00 a month which is 20% of the original salary?

2) The average wage of clerical workers in an insurance company rose 60% during a 10-year period. This increase amounted to $42.50 a week. (i) What was the average weekly salary at the beginning of the 10-year period? (ii) What was the average weekly salary at the end of the 10-year period?

B. BUDGETING

Students need to be made aware of what a budget is and how to make one as most students will have had little experience in budgeting. The percent of income an average person spends on housing, food, entertainment, savings, etc., could be discussed.

The student should be able:

1. to calculate the amount of individual expenditures, given income and the percent rate for each expenditure in a budget;

Examples:

1) The monthly net income of the Johnson family is $1500 and the budget for the family's income is as follows: food, 30%; clothing, 15%; housing, 25%; transportation, 10%; recreation, 15%; savings, 5%. Calculate the amount allocated to each item.

2) Joanne received a $2000 scholarship for her first year at university. Prepare Joanne's budget using the following information: tuition and books, 40%; room and board, 33%; clothing, 14%; recreation, 10%; transportation, 3%.

2. to calculate the percent rate for each expenditure, given the income and the amount of each expenditure in a budget;

Example:

Alvin Shay's net income is $110 per week. Alvin lives at home and has agreed to give his parents $33 per week for his room and board. Other items in his weekly budget include: clothing, $22; recreation, $18; lunches, $12; savings, $25. What percent is budgeted for each item?

Time permitting, teachers may wish to include the following type of example:

Harry Jones saves $200 each month. If Harry saves 10% of his monthly income, what was Harry's income for the month?

3. to prepare a cash budget form, given:

   a) income and expenditures,

Supply students with a copy of a blank cash budget form (see page 109) or have them rule such a form in their notebooks. Discuss and demonstrate the step-by-step method of preparing a cash budget using a form ruled either on a chalkboard or on an overhead transparency. Students should fill in their own forms at the same time. The following information should be included: title, previous balance, entries, period totals, next period's balance, and final totals.

Teachers should emphasize that it is important to manage money wisely by planning and keeping records to show how money is spent. Such plans and records help individuals and families to check waste and to avoid careless spending. It should be pointed out that a cash budget is a plan for the use of one's money.

OBJECTIVES

NOTES TO TEACHERS

Examples:

1) The Jack Smith family prepared a cash budget for the month of May, 1982. It included a balance of $145.25, an estimated income of $960.00 and the following estimated expenses: rent, $350.00; food, $250.00; shoes and clothing, $150.00; recreation, $65.00; miscellaneous items, $25.00; and savings, $50.00. Prepare a Cash Budget Form, showing the totals and estimated balance.

JACK SMITH FAMILY
CASH BUDGET
MONTH OF MAY, 1982

Date		Explanation	Receipts		Disbursements	
		Balance, May 1	145	25		
		Salary	960	00		
		Rent for May			350	00
		Food			250	00
		Shoes and Clothing			150	00
		Recreation			65	00
		Miscellaneous			25	00
		Saving Deposit			50	00
		TOTALS	1 105	25	890	00
		Estimated balance, June 1			215	25

The title, consisting of three lines, identifies the individual or family, the form, and the period of time. The May 1 balance of $145.25, from the preceding month, and the salary Mr. Smith expected to receive during the month were entered in the Receipts column. Each expenditure was entered in the Disbursements column.

After listing the planned expenditures, Mr. Smith completed the cash budget form by totalling each column and then determined the June 1 balance by subtracting.

2) The T. Nugent family prepared a cash budget for the month of October, 1982. It included a balance of $131.40 from September, an estimated income of $850.00 and the following estimated expenses: rent, $325.00; food, $235.00; clothing, $110.00; electric bill, $18.00; recreation, $20.00; savings, $25.00. Prepare a Cash Budget Form, showing the totals and estimated balance.

CASH BUDGET FORM

DATE		EXPLANATION	RECEIPTS		DISBURSEMENTS	

OBJECTIVES

b) income and percent rates of expenditures;

NOTES TO TEACHERS

Example:

The B. Clark family prepared a cash budget for the week of June 1, 1981. The previous week's balance was $167.50. The estimated income was $267.90. The estimated rates of expenditures were as follows: food, 30%; clothing, 15%; housing 20%; transportation, 10%; recreation, 15%; and savings, the remainder. Prepare a Cash Budget Form, showing the amounts of each expenditure, totals and the estimated July 1 balance.

Teachers are encouraged to assign a project in which students prepare a personal budget by entering present income and expenditures on a Cash Budget Form.

Enrichment:

Depending upon student interest and ability and available time, teachers may wish to expand this section to include the preparation of an income and expense record (see blank form, on page 112).

Emphasize the importance of money management. Explain that the success of any budget depends on how closely the actual expenditures correlate with the amounts planned for them. The following sample case could be used to complete an Income and Expense Record:

> Jim Jackson estimates that for the week of November 1, his estimated expenditures will be as follows: savings, $12.50; bus fare and lunches, $10.00; entertainment, $4.00; clothing, $5.00; and other, $3.50. He has a balance of $7.35 from the previous week. His income for the week is $35.00. He puts $12.50 in his savings account. His actual expenses are as follows: November 1, $2.00 for lunch and $0.40 for coke; November 2, $1.95 for lunch and $0.75 for a snack; November 3, $2.00 for lunch and $0.40 for coke; November 4, $2.05 for lunch and $0.30 for a sundae; November 5, $2.00 for lunch and $0.40 for coke; November 6, $2.50 for a show, $4.50 for a belt and $1.10 for a snack; November 7, $1.00 for a snack. Prepare an Income and Expense Record for Jim showing the totals and balance at the end of the week.

OBJECTIVES

NOTES TO TEACHERS

Jim Jackson
INCOME AND EXPENSE RECORD FOR THE PERIOD BEGINNING November 1 ENDING November 7

DATE		AMOUNT OF MONEY RECEIVED			MONEY SPENT OR SAVED		HOW I SPENT MY MONEY														EXPLANATION
							Savings		Bus Fare Lunches		Enter-tain-ment		Clothing		Other						
							EST. AMT. $ 12.50		EST. AMT. $ 10.00		EST. AMT. $ 4.00		EST. AMT. $ 5.00		EST. AMT. $ 3.50		EST. AMT. $		EST. AMT. $		
Nov	1	Balance	7	35																	
	1	Earnings	35	00	14	90	12	50	2	00						40					Coke
	2				2	70			1	95						75					Snack
	3				2	40			2	00						40					Coke
	4				2	35			2	05						30					Sundae
	5				2	40			2	00						40					Coke
	6				8	10					2	50	4	50	1	10					Movie/belt/snack
	7				1	00									1	00					Snack
		TOTALS	43	35	33	85	12	50	10	00	2	50	4	50	4	35					
			33	85																	
Nov	8	Balance	9	50																	

Description:

The title, consisting of two lines, identifies the person, the form, and the period of time.

Note that Jim Jackson divided his spending under the heading How I Spent My Money by writing in the names of the special categories for spending. (These categories may vary from person to person.) Jim used the category, "Other," to record the expenditures that do not fall into special groups. On November 2, for example, he entered 75 cents for a snack in this column. Below each special category, Jim wrote the amount estimated for that category. Putting the Date in the first column and indicating the month at the beginning of each week, Jim recorded his beginning balance (cash on hand) in the Amount of Money Received column. Below the beginning balance, he recorded his earnings (income). In the column, Money Spent or Saved, he entered the total of each day's expenditures ($14.90 = $12.50 + $2.00 + .40).

The totals of the five columns under the heading, How I Spent My Money, should equal the total of the column, Money Spent or Saved ($33.85 = $12.50+$10.00+$2.50+$4.50+$4.35).

OBJECTIVES

NOTES TO TEACHERS

Before the columns are totalled, a single underscore should be drawn to indicate a total. The difference between the totals of Amount of Money Received and Money Spent or Saved should equal the cash on hand (balance) ($43.35 - $33.85 = $9.50). To separate the previous week's Income and Expense Record from the current week's Record, a double line should be ruled. The next week's beginning balance (cash on hand) should be written below the double ruling. The beginning balance (cash on hand) should then be carried over to the next week (or other time period) as money received.

INCOME AND EXPENSE RECORD FOR THE PERIOD BEGINNING ______________ ENDING ______________

DATE	AMOUNT OF MONEY RECEIVED	MONEY SPENT OR SAVED	HOW I SPENT MY MONEY							EXPLANATION
			EST. AMT. $	EST. AMT. $	EST. AMT. $	EST. AMT. $	EST. AMT. $	EST. AMT. $	EST. AMT. $	

Resources:

1. Your Money Matters is a kit consisting of six booklets: You and Your Money, Put Time on Your Side, Planning Your Lifestyle, Value for Your Money, Your Stake in Tomorrow, Your Money Is Their Business. This kit contains colour transparencies and a supporting Guide book. One copy of the kit is free to teachers requesting it on school letterhead. Write:

   The Royal Bank of Canada
   Head Office
   P.O. Box 6001
   Montreal, PQ.
   H3C 3A9

   Or contact: a local branch of The Royal Bank of Canada for information.

2. Lifestyle - Money Management - Budgeting for the Future. Classroom sets of this brochure are free to teachers requesting the material on school letterhead. The package consists of an eight-page brochure and a chart, entitled "Managing Your Money Progress Chart." Write:

   The Canadian Life Insurance Association
   44 King Street West
   Toronto, Ontario
   M5H 1E9

C. CONSUMER PURCHASING

Students should be aware of correct business terminology and terms in common usage. These are not always the same.

Common Usage	Business Term
regular price	regular price
sale price	sale price
amount saved	discount
percent off	rate of discount

A distinction should be made between "on sale" and "for sale." Class discussion could centre on consumer "rip-offs" and "buyer beware," and on the function of the Consumer's Bureau and the Better Business Bureau.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to calculate provincial sales tax;	Three methods could be used: 1) long-hand, using paper and pencil 2) calculator 3) tables Have students practice estimation skills by asking them to estimate sales tax and then find the total cost of purchasing an article. This topic is also discussed in Op.II Basic Skills in Business Mathematics. Indicate that the percent rates of sales tax vary from province to province (5%, 8%, etc.) and that each province applies sales tax to different commodities. Example: How much sales tax will Susan pay when she buys a motorcycle priced at $2595 if the rate of sales tax is 5%? What will be the total amount that Susan must pay for the motorcycle.
2. to calculate unit costs and to make comparisons;	Examples need only include the following type of question. Which is the better buy given that the tins are the same size: - 4 tins of sardines @ 3 for $1.00? - 2 tins of sardines @ 2 for 89¢? Aspects of this objective are dealt with in Op.II Basic Skills in Business Mathematics.
3. to calculate the amount of savings given the sale price and the regular price of an article;	Example: Henrietta buys a record which was priced at $7.95 and is reduced to $6.50. How much did she save by purchasing the record at the reduced price? Solution: Original price $7.95 Less: Reduced price 6.50 Saving $1.45

OBJECTIVES

4. to calculate the amount of savings and the sale price given the discount rate and the regular price of an article;

5. to calculate the percent rate of discount, given the regular price and the amount saved.

NOTES TO TEACHERS

The percent rates used in this section should be realistic -- 10%, 15%, 25%, etc., rather than 8¼%.

Examples:

1) How much does Henrietta save by buying a record, priced regularly at $7.50, at a sale offering a 25% discount? What is the sale price?

Solution:

Original price	$7.50
Less: Discount (7.50 x 25%)	1.88
Sale price	$5.62

2) John buys a sweater which is regularly priced at $12.00 at a sale offering a 20% discount. What is the amount of the discount? What is the sale price?

Solution:

Original price	$12.00
Less: Discount (12 x 20%)	2.40
Sale price	$ 9.60

Example:

The price of a Yamaha snowmobile, regularly priced at $2750, was reduced by $350. What was the percent rate off?

Solution:

Original price	$2750
Less: Discount	350
Sale price	$2400

Rate off = (350 ÷ 2750) = 13%

OBJECTIVES

NOTES TO TEACHERS

For enrichment, the teacher may have students calculate the regular price given the amount of savings and the percent rate off.

Example:

If Hans saved $24.00 by buying a lawn mower at a "20% off" sale, what was the regular price of the lawn mower?

Solution:

Saving = $24.00
Regular price = (24.00 ÷ 20%) = $120.

D. BANKING

The student should be able:

1. to identify and to compare accounts offered by financial institutions including chequing, chequing-savings, and savings accounts;

Provide a description of the three basic types of accounts offered by financial institutions: true savings, chequing-savings, and personal chequing accounts. Teachers could discuss the following: purpose of a true savings account and the current interest rates; purpose of a chequing-savings account and the cost of writing cheques on this type of account; purpose of a chequing account and the cost of writing cheques on this account. Case problems dealing with the need for each type of account could be presented.

In this section, teachers may wish to assign student projects. Suggested topics follow.

a) Services offered by financial institutions

Banks offer these services for a flat monthly charge.

- VISA/Master Card
- unlimited cheque writing on a personal chequing account without the normal service charge
- automatic savings plan
- reduced rate on safety deposit box
- overdraft coverage on VISA/Master card
- multi-banking (interbranch banking)
- free travellers' cheques
- free personalized cheques
- free payment of utility bills and of Videon/Cablevision accounts
- payment of VISA/Master Card
- a "cash card" to obtain cash during or after banking hours (some banks)

OBJECTIVES	NOTES TO TEACHERS

Other Services

Personal Money Order, used inside Canada and issued in Canadian dollars only for amounts not exceeding $500. It is a safe and convenient method of sending money by mail to a person or a business. The Personal Money Order may be redeemed at any branch of the issuing bank or deposited with any other Canadian financial institution.

International Bank Draft, used for sending money outside Canada in domestic or foreign currency. Currency in which International Bank Drafts are issued include Canadian dollars, U.S. dollars, sterling, Italian lire, French francs, German Deutsch marks, Japanese yen, etc. The International Bank Draft is drawn on the bank's correspondent bank for the foreign country.

Domestic Bank Draft, used inside Canada only and issued for amounts over $500 in funds. This draft may be used to send large amounts of money to a person or a business.

b) Cost comparisons between packaged banking services and regular services

Example:

Compare the costs in packaged banking services and in regular banking services for the following:

- safety deposit box
- service charge for writing cheques
- use of travellers' cheques

c) Calculate costs of various accounts given the cost of service charges and packaged banking services

Example:

Jean Smith writes approximately 12 cheques a month. What type of account should Jean Smith use and what will the service charge be?

OBJECTIVES

NOTES TO TEACHERS

2. to write a cheque;

Most students need practice writing cheques.

Note: The Metric Commission Canada recently reviewed the Canadian Bankers' Association concerns on the matter of triad separators for monetary values and indicated that the practice of using a space as a triad separator "need not apply to monetary values and shall not be used on payment instruments." This guide will continue to express numbers such as 56,384 as 56 384, 3.14159 as 3.141 59, and $27,464.09 as $27 464.09. However, on all payment instruments students should use commas rather than spaces as triad separators to avoid the possibility of fraudulent practice.

3. to calculate balances on chequing accounts, given the amount of deposits and cheques;

Provide students with case problems on balancing a chequing account. Calculate solutions to problems on a cheque book record form similar to the following:

CHEQUE NO.	DATE	CHEQUE ISSUED TO	AMOUNT OF CHEQUE		✓	DATE OF DEP.	AMOUNT OF DEPOSIT		BALANCE	

OBJECTIVES

4. to update a cheque book journal or register given the amounts of cheques, deposits and service charges;

5. to reconcile a bank statement.

NOTES TO TEACHERS

Example:

Judy Bradley has a personal chequing account with a beginning balance of $1200. Using the following information, calculate the end-of-the-month balance in Judy's personal chequing account:

> Judy wrote a cheque to Bright Dry Cleaners for $15.30 on May 5 and one to ABC Grocery Store for $63.86 on May 7. The bank service charge for the month was $0.50.

Include problems with negative balances.

Jerry Callan has a personal chequing account with a beginning balance of $150.00. He wrote the following cheques: $75 to Krazy Carpets, $60 to ABC Grocery, and $83 to The Village Shoe Store. What is the ending balance?

At this point the teacher could introduce the students to two methods of handling the negative balance:

1) The bank will charge an overdraft charge for an overdrawn account. To continue with the previous problem, the student should deduct $3 for an overdraft charge.

2) If the bank customer used packaged banking services, the bank may cover the overdrawn amount by obtaining a cash advance from the customer's VISA/Master Card. In the preceding problem, the student should add $70 to the customer's account. The customer now owes $70 to VISA/Master Card who charge the customer the going interest rate on the cash advance.

Teachers must emphasize the importance of immediately recording cheques and deposits as they are made.

Stress that monthly reconciliation is imperative for keeping accurate accounts.

A bank reconciliation form is used to calculate the amount of money available in a chequing account. This amount should agree with the customer's cheque book balance. Students should understand the purpose of numbering cheques.

Following are examples of two types of bank reconcilation forms.

OBJECTIVES

NOTES TO TEACHERS

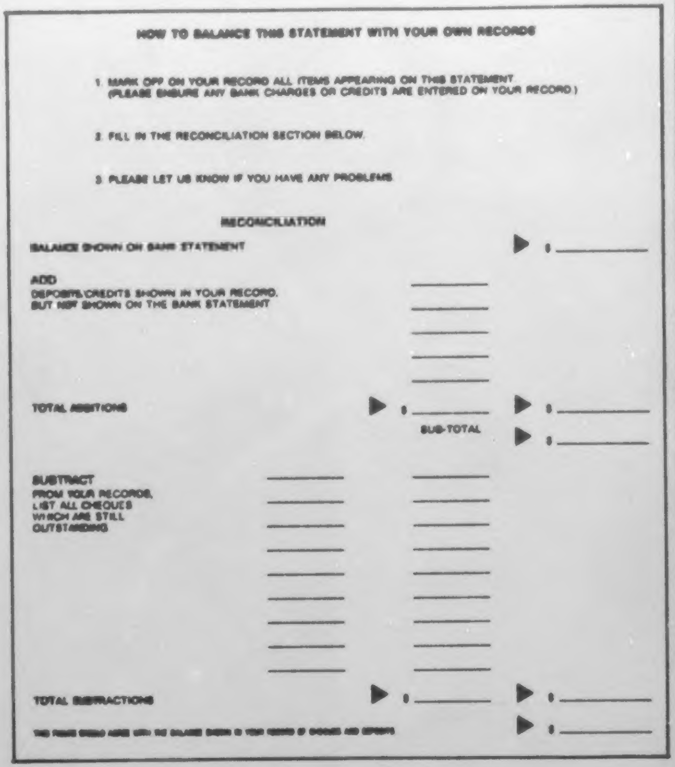
Reconciliation Statement

________________19__

A. Outstanding cheques NO./DATE	AMOUNT	
1.		
2.		
3.		
4.		
TOTAL (A)		
B. Closing Balance shown on the bank statement		
C. PLUS deposits made after last entry on this statement		
D. Total (B+C)		
E. MINUS total of outstanding cheque(s) (A)		
F. Your bank balance (D-F)		
G. Chequebook balance		
H. Service charge		
I. Correct chequebook balance (G-H)		

OBJECTIVES

NOTES TO TEACHERS



HOW TO BALANCE THIS STATEMENT WITH YOUR OWN RECORDS

1. MARK OFF ON YOUR RECORD ALL ITEMS APPEARING ON THIS STATEMENT.
   (PLEASE ENSURE ANY BANK CHARGES OR CREDITS ARE ENTERED ON YOUR RECORD.)

2. FILL IN THE RECONCILIATION SECTION BELOW.

3. PLEASE LET US KNOW IF YOU HAVE ANY PROBLEMS.

RECONCILIATION

BALANCE SHOWN ON BANK STATEMENT ▶ $ ______

ADD
DEPOSITS/CREDITS SHOWN IN YOUR RECORD,
BUT NOT SHOWN ON THE BANK STATEMENT

TOTAL ADDITIONS ▶ $ ______ ▶ $ ______

SUB-TOTAL ▶ $ ______

SUBTRACT
FROM YOUR RECORDS,
LIST ALL CHEQUES
WHICH ARE STILL
OUTSTANDING

TOTAL SUBTRACTIONS ▶ $ ______ ▶ $ ______

THIS FIGURE SHOULD AGREE WITH THE BALANCE SHOWN IN YOUR RECORD OF CHEQUES AND DEPOSITS ▶ $ ______

OBJECTIVES

NOTES TO TEACHERS

Example:

Prepare a bank reconciliation statement for Mary Strachan from the following information: Mary's present cheque book balance is $315.77. She received a bank statement indicating the following: a bank balance of $300.20 and a service charge of $3.50. Mary discovered that the following cheques were outstanding:

#202 - $ 8.91
#210 - $14.50
#212 - $22.88

She also discovered that deposits in the amount of $150.00 and $50.00 were not included in her bank statement.

Resources:

1. A Student Guide to Banking - Students' Portfolio

   This portfolio includes blank bank forms and a short history of banking. Classroom sets of the portfolio in English or French are free to teachers requesting them on school letterhead.

2. A Student Guide to Banking - Teacher's Instruction Booklet

   The Teacher's Instruction Booklet, a manual for a Student Guide to Banking, includes completed forms with detailed instructions. One copy of this booklet in English or French is free to the teacher requesting it on school letterhead.

Remember to order both of these resource materials in advance of the time required.

Write: Canadian Imperial Bank of Commerce
Head Office
Commerce Court
Toronto, Ontario
M5L 1G9

Or contact: A local branch of the Canadian Imperial Bank of Commerce.

Note: Teachers may wish to use the microcomputer to reconcile a bank statement or to reinforce the previous lessons. Interested teachers could obtain the "Bank Reconciliation" program from Micro Learningware, P.O. Box 2134, North Mankato, Minnesota, 56001, for use with Commodore PET, or Radio Shack TRS 80 microcomputers.

C.III. GEOMETRY (1½ modules)

Geometry in 101 Mathematics is to be taught informally. There are no proofs to be learned, no formulae to be derived or memorized. When a formula is necessary, it should be provided. Discussions of geometry in the environment can be a means of reviewing vocabulary. Students could identify such geometric forms as the rectangle (blackboard), the circle (clock face), the sphere (baseball), the cone (ice cream cone), and the triangle (traffic yield sign). Examples could be chosen from school, home, or business.

Students should be encouraged to use calculators once they have understood the concepts and algorithmic processes involved in this module and are able to solve problems on their own. They should also, however, know how to estimate and to check the reasonableness of their answers. Refer to Op.IX Estimation.

Circles will be taught in the Mathematics 201 Geometry core unit. Concepts involving the circle are not intended to be part of Math 101.

OBJECTIVES

A. PERIMETER

The student be able:

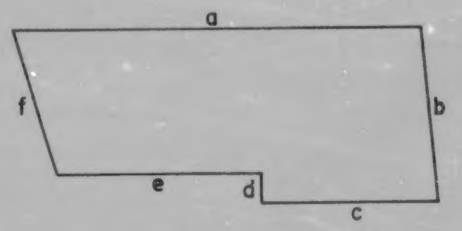
1. to demonstrate an understanding of the concept of perimeter as a measure of the distance around;

NOTES TO TEACHERS

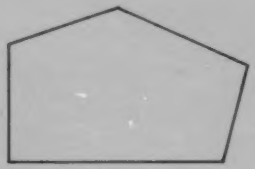
It is intended that the approach be informal, experimental, and intuitive. Both irregular and regular shapes should be measured. Students could be asked to estimate the length of the baseboard in the classroom, the length of a string of Christmas lights for a window, or the length of the flood dike around their town.

For additional suggestions on estimation/approximation see Op.IX Estimation.

Example:



Estimate the number of centimetres in the lengths a, b, c, d, e and f. Now measure them. How accurate were your estimates? What is the actual perimeter of the figure?

OBJECTIVES	NOTES TO TEACHERS
2. to state the common metric units of perimeter (mm, cm, m, km) and to convert from one unit of linear measure to another;	Teachers are expected to use only the units of measure identified in the objective. Example: Which unit would you use: mm, cm, m or km? 1) The perimeter of Lake Winnipeg is about 1000 ____. 2) The distance around your desk top is about 175 ____. 3) The perimeter of a dollar bill is about 440 ____.
3. to determine the perimeter of a polygon when given the lengths of the sides and when measurement is required;	Exercises should be provided to find the perimeter of squares, rectangles and triangles as well as other polygons. Example: Determine the perimeter of the accompanying figure.  Solution: Measure the length of each side. Then add.
4. to solve word problems involving perimeter.	Emphasis should be placed on the translation of word descriptions into mathematical phrases, with problems posed in such a way that the student is required to think about how to solve them. They should not be exercises designed solely for reinforcing arithmetic skills. Applications should involve practical examples and should use correct SI symbols. Applications could include finding the length of fence for a flower bed, the length of ribbon to wrap a parcel, or the length of wood to make a picture frame. Students should be encouraged to use calculators. Example: Judy helped build a fence around a new rectangular corral. The corral measures 22.6 metres long by 14.9 metres wide. What is the perimeter of the corral?

OBJECTIVES

B. AREA

The student should be able:

1. to demonstrate an understanding of the concept of area as the number of unit squares on a surface;

2. to state the common metric units of area (mm^2, cm^2, m^2, km^2) and to convert from one area measure to another;

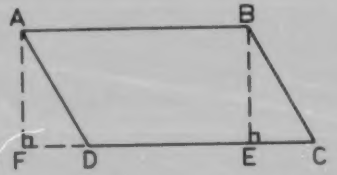
3. to determine the area of a square, rectangle, parallelogram, triangle, trapezoid and rhombus through computation or through measurement using squared paper;

NOTES TO TEACHERS

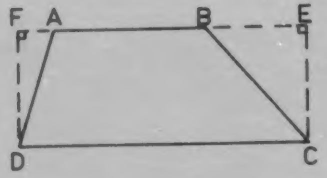
Concrete activities should be provided. Students could draw figures on squared paper (graph paper) and count the squares, or they could draw squares over a figure on paper and then count the squares.

Students must realize that the units for measuring area have two dimensions; they are square units. Also, students must be aware that measuring area implies measuring the interior of a figure, while perimeter measures only the edge.

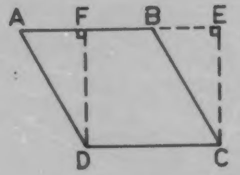
The concept of the area of a square and of a rectangle could be developed by counting squares. Ultimately, students are to realize that area is the number of rows times the number of square units per row. The area of a parallelogram, trapezoid and rhombus could be developed by showing how each figure can be converted into a rectangle or a square.



If ΔBEC is removed and placed along side AD, a rectangle ABEF is formed which is the same area as parallelogram ABCD.



The area of trapezoid ABCD can be found by subtracting the area of ΔFAD and ΔBEC from the area of rectangle FECD.



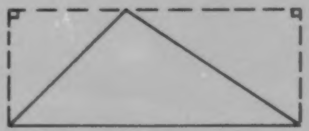
The area of rhombus ABCD can be shown to be the same as the area of the rectangle FECD.

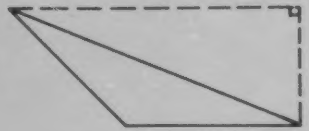
OBJECTIVES

4. to solve word problems involving area.

NOTES TO TEACHERS

Similarly, the concept of the area of a triangle could be developed using a rectangle or a parallelogram.





It must be stressed that unless the base measure and height measure are perpendicular to each other, the basic unit of square measure cannot be used.

The formula for each of the areas should be introduced only after students have developed an understanding of the concept of area. Formulae are not to be memorized. Students should know how and where to find the required formulae in textbooks.

Emphasis is to be placed on translating word statements into mathematical symbols and on deciding correct computational procedures. Applications could include finding the area of a football field, determining the quantity of paint required for a bedroom, or determining the quantity of carpet or tile required for a floor.

Example:

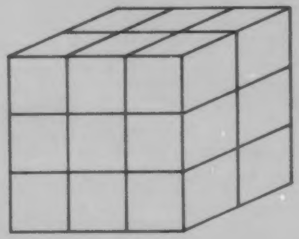
A basement ceiling is 5.6 m wide and 9.4 m long. How many boxes of tiles are needed to cover the ceiling if each box covers $4 m^2$?

Solution:

Area = $5.6 \times 9.4 = 52.64\ m^2$.

Each box of tiles covers $4\ m^2$.

14 boxes of tiles are required.

OBJECTIVES	NOTES TO TEACHERS
C. VOLUME The student should be able:	
1. to demonstrate an understanding of the concept of volume as the number of unit cubes in a given solid;	Since students have difficulty working with three-dimensional solids, they should be given concrete activities to begin with. Non-standard units may be used to introduce the concept. Examples could include the number of sugar cubes in a box or the number of tins of juice used to fill a case. Example: Determine the volume of the following figure: 
2. to see that in some cases (cube, rectangular solid) the volume is equal to the area of the base times the height;	Students may discover that volume is equal to the number of cubic units in one layer times the number of layers; in other words, the area of the base times the height.
3. to state the common metric units of volume (mm^3, cm^3, m^3, km^3) and to convert from one volume measure to another;	Oceanographers use the unit km^3 to measure currents and volume of water.

OBJECTIVES

4. to determine the volumes of rectangular solids, when given the lengths of the sides and through measurement;

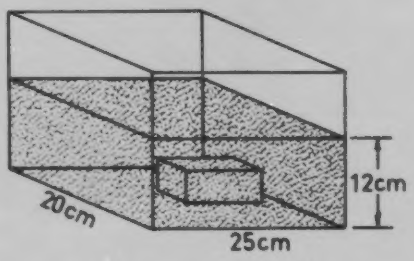
5. to solve word problems involving volume.

NOTES TO TEACHERS

Applications must involve real world examples such as finding the number of cubic metres of sand a truck box can hold, the number of metres of dirt removed from a basement excavation, or the amount of water needed to fill a fish tank.

Example

Before the brick was placed in the tank, the depth of the water was 9 cm. What is the volume of the brick?



D. APPLICATION OF PERIMETER, AREA, AND VOLUME

The student should be able:

1. to solve word problems involving perimeter, area, and volume by recognizing the type of measurement required and then making the appropriate calculations;

Students are encouraged to use calculators. Real world applications could involve finding the area of carpet for a given floor plan, the amount of fertilizer needed for a lawn, or the volume of a swimming pool with a sloped bottom. Estimating the area of an irregular figure and finding the area of a figure that can be reduced to a sum of rectangles could also be attempted. Volumes of rectangular solids should also be used.

OBJECTIVES

2. to recognize and to distinguish between the units of measurement for perimeter, area, and volume;

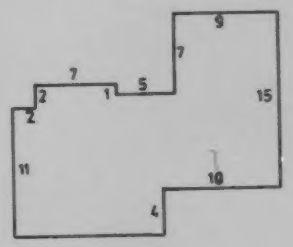
3. to determine a missing dimension given the perimeter, area, or volume and the other dimension(s).

NOTES TO TEACHERS

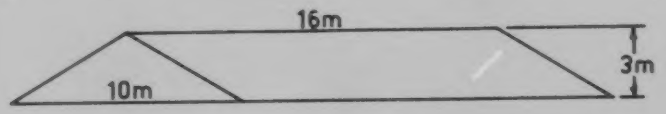
Examples:

1) A property is 413 m long on the north side, 1666 m long on the east side, 1719 m long on the west side, and 400 m long on the south side. What length of fence is needed? Allow 7 m for a gate opening.

2) Find the area of the accompanying figure if all measurements are in centimetres.



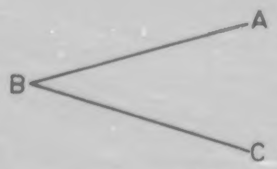
3) Find the volume in cubic metres of a house attic, dimensions of which are shown in the sketch.



Only simple examples are to be used.

Example:

The area of a rectangle is 60 cm^2. Its length is 12 cm. Find its width.

OBJECTIVES	**NOTES TO TEACHERS**
E. CAPACITY The student should be able: 1. to recognize the common metric units of capacity (mL, L); 2. to convert from one capacity unit to another.	Fluids are usually measured in litres or millilitres whereas solids are measured in cm^3. Please note that the symbol for litre is L and for millilitre is mL. Example: If Super-Duper Cleaning Fluid costs 95¢ per 500 mL how much would John have to pay for 3.5L of fluid?
F. MASS The student should be able: 1. to recognize and to state the common metric units of mass (mg, g, kg, t); 2. to convert from one mass unit to another.	Applications could involve grocery or drug store purchases and unit pricing. A scale could be used for measuring. Example: A candy bar has a mass of 39 g and costs 35¢. A larger bar costs 95¢ and has a mass of 88 g. Which is the better buy? Discussion could revolve around units for measuring water. Students should realize that 1000 cm^3 of water has the same mass as 1000 mL of water or 1 kg of water at 4°C.
G. IDENTIFICATION OF ANGLES The student should be able to identify the sides and the vertex of an angle, and to name an angle.	Students should realize that an angle represents a rotation of the terminal arm from the initial arm.  This angle can be named ∠ABC, or ∠B, or ∠CBA.

OBJECTIVES

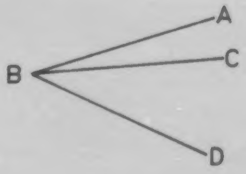
H. ANGULAR MEASUREMENT

The student should be able:

1. to recognize the units of angular measurement as degrees or fractions of a revolution;

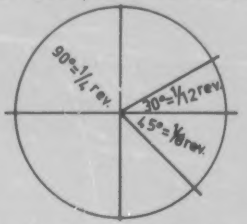
2. to measure angles from 0° to 360° in either revolutions or degrees (to the nearest degree);

NOTES TO TEACHERS



In this case one cannot speak of ∠B since there are three angles at B in the diagram: ∠ABC, ∠ABD and ∠CBD. One could also use numbers to name angles.

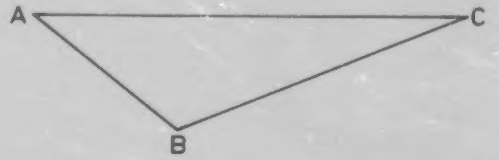
A circle is made up of 360 parts called degrees. Thus, one revolution has 360°.



Students have a great deal of difficulty measuring angles. It is essential, therefore, that they be given practice using a protractor. They also must realize that the size of an angle is independent of the length of the arms of the angle.

Examples:

1) Measure the angles in the accompanying triangle.



OBJECTIVES	NOTES TO TEACHERS
	2) A wheel has 5 equally spaced spokes. What is the measure of the angle between the spokes? Solution: Angle per spoke = $\frac{360°}{\text{no. of spokes}} = \frac{360}{5} = 72°$
3. to draw with a protractor any angle whose measure is less than or equal to 360° or 1 revolution; 4. to identify angles according to their size as being acute, right, obtuse, straight or reflex.	

C.IV. ALGEBRA (1½ modules)

Many of the difficulties that students experience in the study of algebra may be attributed to the higher degree of generalization and abstraction in this area of mathematics than in others encountered previously. In contrast to arithmetic, algebra deals more with a conscious examination and study of basic structure than with answers to particular problems. Many mathematics teachers will agree on the importance of proper motivation in the teaching of algebra. With the exception of the very few students who seem to catch on quickly and exhibit an immediate understanding, most others need to be stimulated through suitable teaching techniques and procedures. Any aids which can help make algebraic concepts more explicit deserve attention.

Teachers and students could discuss reasons for a study of algebra. Practical real-life applications for algebraic concepts should be sought. Encourage students to recognize the practical uses and applications of algebra in the community by introducing practical application problems that involve algebraic concepts.

Teachers are advised to read the objectives carefully because they contain restrictions on the algebra to be covered. Extension of the objectives may result in other topics in the course not being covered properly.

OBJECTIVES	NOTES TO TEACHERS
A. EXPONENTS, SQUARE ROOTS AND SUBSTITUTION The student should be able:	
1. to explain and to illustrate the meaning of the algebraic terms: a. base, b. exponent, c. power, d. coefficient;	Students should not be required to memorize meaningless definitions. However, all students need a working knowledge of the terminology associated with the discipline of mathematics. Example: In the algebraic term x^4, "x" is the base, "4" is the exponent and "x^4" is the power; whereas, in $6x^4$, the "6" is the numerical coefficient or coefficient. Students must realize that x^4 is the same as $1x^4$. Students should be aware that there are both numerical and literal coefficients even though most exercises will involve only numerical coefficients.

OBJECTIVES

2. to convert a power with a natural number exponent and a rational base into a simple real number;

3. to convert an integer into exponential form;

NOTES TO TEACHERS

Since students often have difficulty understanding and manipulating algebraic powers, they must spend time manipulating numerical powers and expressions. Students should become proficient in the types of questions listed below.

Examples:

Determine the exact numerical value of:

1) 6^3

2) $(1.2)^2$

Solution:

1) $6^3 = 6 \times 6 \times 6$ or $(6)(6)(6)$

$= 36 \times 6$

$= 216$

2) $(1.2)^2 = 1.2 \times 1.2$

$= 1.44$

Note: If teachers or textbook authors use "x" for a variable, then it is suggested that brackets be used for multiplication.

Students have difficulty differentiating between $4^2 = 16$, $(-4)^2 = 16$, $-4^2 = -16$, and $-(4)^2 = -16$. These distinctions are necessary when evaluating formulae by substitution.

Examples:

$49 = (7)(7) = 7^2$ or $(-7)(-7) = (-7)^2$

$-27 = -(3)(3)(3) = -3^3$ or $(-3)(-3)(-3) = (-3)^3$

$-24 = (-3)(8) = (-3)(2)(2)(2) = (-3)(2^3)$, etc.

OBJECTIVES	NOTES TO TEACHERS
4. to understand the meaning of square root and to recognize the symbol for square root;	Square roots of perfect squares up to 144 may be used for concept illustrations. Students should realize that squaring means multiplying a number by itself ($3^2 = (3)(3) = 9$), whereas finding a square root means finding that number which mutliplies by itself to yield the desired result ($\sqrt{9} = 3$ since $3 \cdot 3 = 9$). Note: The concept of square root should be taught carefully. The principal square root, by agreement, is $\sqrt{9} = 3$ and not both +3 and -3. However, students must realize that $-\sqrt{9} = -3$. Most North American authors use $\sqrt{\ }$ to represent square root. This is actually a combination of two separate symbols $\surd$ (square root) and $\overline{\ }$ (vinculum, a grouping symbol).
5. to find the square root of numbers using either calculators or square root tables;	Students are not expected to calculate square roots long-hand. Students must realize that the computation of the square root of a number which is not a perfect square is approximate and not exact. For students who do not have a square root function on their calculators, a table should be provided.
6. to evaluate given formulae by substitution.	Wherever possible, use examples that have realistic and practical applications and that are interesting to students, in this way avoiding sterile expressions such as $y = 3x^2 - 2xp + t^3$. Assume that mathematical applications abound in the immediate environment. But teachers are busy preparing, marking and evaluating, so time to ferret out these applications may be lacking. Perhaps students could be assigned a project in which they are to show the practical everyday use of mathematics in a local business or occupation. Word and diagram problems could be used as long as necessary formulae are provided. This is a more difficult exercise involving the recognition of values for specific variables and then substituting those values into the given formula. Teachers may prefer to integrate this work with perimeter, area, volume, etc. Teachers are reminded that some explanation of terminology will need to be provided for each mathematical application.

OBJECTIVES

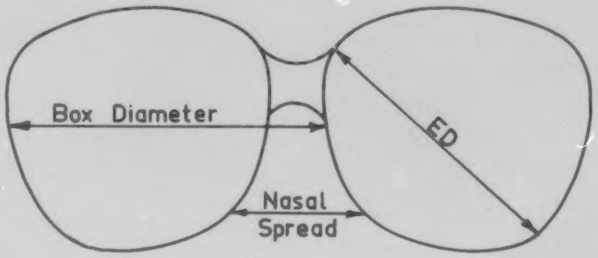
NOTES TO TEACHERS

Example 1:

The thickness of the lens in an eyeglass depends upon the kind of frame chosen, the distance between the pupils, and the optometrist's prescription. Clinicians determine lens thickness by using the following formula:

$$T = [(\tfrac{1}{2}ED + TD)^2]\, R_x \div 1000$$

where T = lens thickness; ED = effective diameter (measurement from top nasal part to bottom temple part of frame); TD = total discentration (difference between the sum of box diameter and nasal spread, which are numbers found on the frames, and the pupil distance); and R_x = optometrist's prescription.



Sample problem:

Determine the lens thickness if the pupillary distance is 62 mm, the box diameter is 51 mm, the nasal spread is 19 mm, the effective diameter is 66 mm, and the optometrist's prescription is 3.00.

$$T = [(\tfrac{1}{2}ED + TD)^2]\ R_x \div 1000$$

$$= [(\tfrac{1}{2}(66) + 8)^2 \cdot 3.00] \div 1000$$

$$= 5.043$$

The lens is 5.043 mm thick in this example.

Source: T. Monk from M. Bilicki, House of Spectacles, Garden City Shopping Centre, Winnipeg, Manitoba.

Example 2: Accounting Equation

The accounting equation contains three fundamental elements: assets, liabilities, and owner's equity. The assets represent what a business owns (students should be able to supply examples). Liabilities and the owner's equity represent the total claims against the assets. Liabilities are the claims of creditors. The owner's equity is the claim of the owner. (Whatever is not claimed by the creditors belongs to the owner.) Total claims (liabilities and owner's equity) against the assets are always equal to the total assets. The relationship between assets and liabilities is expressed by the accounting equation:

ASSETS = LIABILITIES + OWNER'S EQUITY, or

$$A = L + E$$

Note: Students should realize that accounting texts may use O.E. for Owner's Equity.

OBJECTIVES

B. POLYNOMIAL OPERATIONS

The student should be able:

1. to identify and to collect like terms;

NOTES TO TEACHERS

Example:

Give the amount missing from each accounting equation:

Assets A	=	Liabilities L	+	Owner's Equity E
\$10 000	=	\$1000	+	?
2 800	=	250	+	?
?	=	1500	+	\$6500
?	=	2200	+	8000
10 200	=	?	+	5800
8 320	=	?	+	5200

Other examples could include:

Net Income = Revenue - Expenses

Net Price = List Price - Trade Discount

Selling Price = Cost + Markon

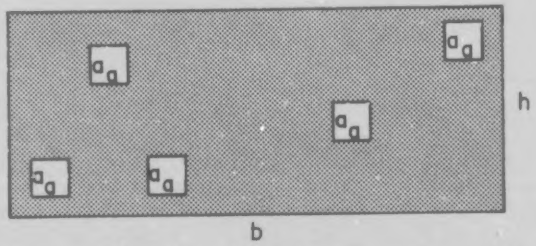
Interest = Principal x Rate x Time

Teachers are reminded to use terminology appropriate for students. Students should be familiar with terms such as monomial, binomial, like terms, variable, polynomial, etc.

Example:

Simplify by combining like terms: $bh-a^2-a^2-a^2-a^2-a^2$

One model demonstrating the process of collecting like terms is illustrated below.



Determine the area of the shaded region of the above diagram.

Solution:

$$\begin{aligned}\text{Shaded area} &= bh-a^2-a^2-a^2-a^2-a^2\\ &= bh-5a^2\end{aligned}$$

Note: The collection of like terms leads to a simpler and/or shorter expression.

2. to add or to subtract algebraic expressions;

Students should be prepared to perform addition and subtraction of algebraic expressions either horizontally or vertically.

Examples:

Perform the indicated operation(s) on $(2x^2 + 5xy) + (3x^2 - xy)$ or $(2x^2 + 5xy) - (3x^2 - xy)$.

$$\text{add}\quad \begin{array}{r} 2x^2 + 5xy \\ \underline{3x^2 - xy} \\ \underline{} \end{array} \qquad \text{subtract}\quad \begin{array}{r} 2x^2 + 5xy \\ \underline{3x^2 - xy} \\ \underline{} \end{array}$$

3. to multiply and to divide monomials by integers and decimal fractions;

These exercises are designed to make future work on formula manipulation easier.

Examples could include:

$(3x^2)(2)$; $(6x^2)\div(-3)$; $(5x^2)(0.24)$; $(4.7ab)\div(0.047)$, etc.

Answers to questions such as $(2x^2)\div(3)$ could be written as $\frac{2x^2}{3}$ or $\frac{2}{3}x^2$ or $0.\overline{6}x^2$, where $\overline{6}$ means 6 repeating.

OBJECTIVES	NOTES TO TEACHERS
4. to multiply and to divide algebraic expressions by integers and decimal fractions;	Examples should proceed from $2(5x-1)$ to $-7(5a-b+1)$ to $-(8t-4)$ to $0.2(5x-4)$. This objective does not include examples such as $a(b+2)$ or $4x(3x-y)$, which are included in Mathematics 201.
5. to simplify algebraic expressions involving addition, subtraction, and multiplication.	Students are expected to simplify by multiplying to remove brackets, and then to combine like terms in such examples as: $2(5x-1) + 3(x-4)$ or $2(5x-1) - 3(x-4)$.
C. SOLVING EQUATIONS The student should be able:	
1. to write an equation when given the solution;	Students should be familiar with solving equations to the extent that they realize that "what you do to one side you do to the other." Have students build a complex equation. Example: Begin with $x = 2$ and build a more complex equation. $x = 2$ $x - 5 = 2 - 5$ subtract 5 from both sides $\frac{x-5}{3} = \frac{2-5}{3}$ divide both sides by 3 $\frac{x-5}{3} + 1 = \frac{2-5}{3} + 1$ add 1 to both sides $\frac{x-5}{3} + 1 = 0$ combine numbers on the right side After several such operations, ask students to determine the solution. (Answer $x = 2$) It is critical that students know what an equation tells them to do and in what order it tells them to do it since the reverse procedure is necessary to solve the equation. Example: Translate $\frac{x-5}{3} + 1 = 0$ either verbally or in writing.

2. to solve linear algebraic equations in one variable;

Solution:

From some number "x" subtract "5," divide this result by "3" and then add "1" in order to obtain "0."

It is suggested that the variable be restricted to one side of the equation in this type of exercise.

Students are encouraged to solve equations by analyzing as in the previous section and by applying inverse operations in the opposite order.

Sequence of development:

1) $x + 3 = 5$

2) $5x = 7$

3) $4y - 7 = 9$

4) $3t - 7 = t + 1$

5) $4(5x-3) = -8$

6) $4(x+7) - 3(2x-1) = 5$

7) $4(d-5) + 3(2d-1) = -(3d-2)$

8) $\frac{x+3}{4} = \frac{x+5}{6}$

Analyzing the order in an equation, then performing the operations in reverse order (using inverse operations) is one method for solving equations.

Example:

Solve for x.

$4(5x-3) = -8$

OBJECTIVES

3. to solve simple quadratic equations of the form $ax^2 = c$ where "a" and "c" are constants with the same sign and $a \neq 0$;

NOTES TO TEACHERS

Solution:

Equation Says	Solving Means (Reverse-Inverse to what equation says)	Worked Solution
		$4(5x-3) = -8$
- multiply x by 5	- dividing both sides by 4	$\frac{4(5x-3)}{4} = \frac{-8}{4}$ $5x-3 = -2$
- subtract 3	- adding 3 to both sides	$5x-3+3 = -2+3$ $5x = 1$
- multiply by 4 to get -8	- dividing both sides by 5	$\frac{5x}{5} = \frac{1}{5}$ $x = \frac{1}{5}$ or 0.2

Students should know how to check solutions to equations, although they should not be expected to show a written check for each question. However, students need to check some examples in order to show the method.

Example:

Solve for "y" in $2y^2 = 18$

Solution:

$2y^2 = 18$

$\frac{2y^2}{2} = \frac{18}{2}$

$y^2 = 9$

$y = \sqrt{9}$ or $y = -\sqrt{9}$

$y = 3$ or $y = -3$

Note: The use of the notation $y = \pm 3$ is not encouraged.

OBJECTIVES

4. to express word phrases as algebraic expressions and vice-versa;

NOTES TO TEACHERS

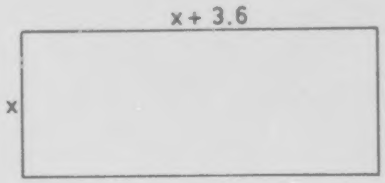
Students need considerable practice in translating from word phrases to algebraic expressions. An attempt should be made to use practical examples from business, trade and vocational areas.

Examples:

1) What are the dimensions of a rectangular office building if the length is 3.6 m more than the width?

Solution:

Let the width be x metres; the length would then be (x+3.6) metres.

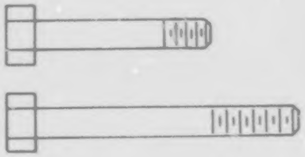


Dimensions are x metres by (x+3.6) metres.

2) Express, in algebraic form, the lengths of two stove bolts if the longer must be 1.5 cm less than twice the length of the shorter bolt.

Solution:

Consider the following sketches:



Let the shorter bolt be "y" cm in length.

Then twice the shorter bolt is expressed as "2y" cm and 1.5 cm less than twice the length of the shorter bolt is written as (2y-1.5)cm.

Lengths of bolts are y cm and (2y-1.5) cm.

OBJECTIVES

5. to translate word problems into algebraic equations and vice-versa.

NOTES TO TEACHERS

The emphasis here is on creating equations and not on solving them. Students should use any letter they choose, but should always identify what the letter or variable represents.

Examples:

1) Translate into an algebraic expression:

The area of a trapezoid is one-half the height multiplied by the sum of the two bases.

Solution:

$A = \frac{1}{2}h(B+b)$ where A is the area of the trapezoid, h is height, B and b represent the length of the bases.

2) Write a word statement for the expression $S = B+C$ where S is salary, B is basic wage and C represents commission.

Solution:

Salary is the sum of the basic wage and the commission.

CALCULATOR MATHEMATICS

Op.I. OPERATIONS OF AND APPLICATIONS FOR CALCULATORS (½ module)

This unit may be taught in conjunction with, or in addition to, C.I Arithmetic, and may be interspersed throughout the course. Each calculator has a specific sequence of operations and for this reason, specific steps for solutions have not been included.

Students should be encouraged to estimate their answers prior to performing calculations. Further, they should be encouraged to write out the order of calculation for problems or exercises that involve several operations.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to perform on the calculator the basic operations of addition, subtraction, multiplication and division;	Students should be shown that incorrect error-corrections can sometimes be made if the calculator is not cleared completely.
2. to solve problems using the correct order of operations;	Students should be reminded that the correct order of operations is brackets, multiplying - dividing, adding - subtracting. Examples: 1) $4 + 7 \times 3 - 2 = 23$ 2) $\frac{6}{2\times3} = 1$
3. to convert common fractions to decimal fractions;	There should be an agreement as to the rounding procedures used. Teachers may prefer answers written to, say, 3 decimal places. Examples: 1) $\frac{4}{9} = .444$ 2) $\frac{2}{3} = .667$

OBJECTIVES	NOTES TO TEACHERS
4. to perform calculations involving percent;	Example: Calculate 15% of 48 = ? Answer = 7.2
5. to perform operations involving square roots and reciprocals;	Students may learn to appreciate the square root concept by working through the following activity, which involves determining the square root of any given number without using √ function. Example: Calculate the square root of 498 using trial and error. Solution: Guess 15: 15 x 15 = 225 (15 is too low). Now try 25: 25 x 25 = 625 (25 is too high). Trial and error leads to pursuit of an answer between 15 and 25.
6. to use the memory function;	Students should be made aware that the memory function can expedite certain calculations. Example: Find the value of $\frac{(7.9+5.8)(8+1.3)}{4}$ Solution: Calculate (7.9+5.8); store the result in memory; calculate (8+1.3); multiply by memory; divide by 4.
7. to recognize and to interpret unusual displays.	Students should be made aware of unusual displays such as: 1) error 2) scientific notation 3) weak power source 4) rounding of very large numbers.

Op.II. BASIC SKILLS IN BUSINESS MATHEMATICS (1 module)

A. RAPID CALCULATION

The main thrust of this section is to firm up a student's arithmetic skills. Both speed and accuracy are to be considered. Daily drills for five to ten minutes of rapid calculation (mental as well as paper and pencil calculation) should be provided. Use timed drills (one minute, two minutes, five minutes, ten minutes) for each of the objectives. Since drill work is more effective when integrated throughout the entire program, teachers should consider interspacing this unit throughout the school term.

The following materials contain suggestions and exercises for rapid calculation:

1) "Rapid Calculation for Business Mathematics"
"Package for Rapid Calculation and Preliminary Drills"

Available from: Curriculum Services
404 - 1181 Portage Avenue
Winnipeg, Manitoba
R3G 0T3

2) Rapid Calculation and Estimating Skills by A. Park and L. Racknor available from Copp Clark Pitman. This fully metric publication has two divisions: Part 1 - Rapid Calculation and Part 2 - Estimating Skills.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to identify the place value of each digit from a thousandth to a million;	Example: Identify the place value of the 6 in each of the following: 1) 632.51 2) 1 549 832.263
2. to read and to write numbers between thousandths and millions;	Example: Write in words: 1) 526.93 2) 61 985

OBJECTIVES | **NOTES TO TEACHERS**

3. to add a column of numbers;

Suggested shortcuts are:

1) grouping by tens
2) adding 2 columns at the same time

Example:

$$\begin{array}{r} 57\\ 36\\ +\ 23\\ \hline \end{array}$$

Starting from the bottom, add 23 and 6, then, add 29 + 30; then add 59 and 7, then add 66 and 50. The result is 116.

4. to add, to subtract, to multiply, and to divide whole numbers;

5. to add, to subtract, to multiply, and to divide decimal fractions;

This is an important section. Have students use common decimal fractions such as 0.5, 0.25, 0.4, 0.75, or 0.05.

6. to multiply by 0.001, 0.01, 0.1, 10, 100, 1000 by moving the decimal point;

7. to divide by 0.001, 0.01, 0.1, 10, 100, 1000 by moving the decimal point;

Have students realize that dividing by 0.01 and multiplying by 100 results in the same answer.

8. to convert one type of fractional number to another:
   a. common fraction to decimal fraction,
   b. common fraction to percent rate,
   c. decimal fraction to percent rate,
   d. percent rate to decimal fraction;

Example:

Fill in the blanks:

Common Fraction	Decimal Fraction	Percent
½		
	.25	
		40%

OBJECTIVES

NOTES TO TEACHERS

Students are not expected to convert decimal fractions or percent rates to common fractions.

Use common fractions such as these: $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, and $\frac{1}{10}$.

Use decimal fractions such as these: 0.125, 0.375, 0.625, 0.875, $0.33\overline{3}$, $0.66\overline{6}$, 0.5, 0.25, 0.2, 0.4, 0.6, 0.8, and 0.1.

Use percent rates such as these: $12\frac{1}{2}$%, $37\frac{1}{2}$%, $62\frac{1}{2}$%, $87\frac{1}{2}$%, $33\frac{1}{3}$%, $66\frac{2}{3}$%, 50%, 25%, 20%, 40%, 60%, 80%, and 10%.

9. to find the percentage of a number given the percent rate and the number;

Example:

Find 20% of 75.

10. to calculate the price of a quantity of articles if the price of one is known;

Example:

Find the cost of 3 shirts if the cost of each is $11.98.

Solution:

Round off 11.98 to $12

$12 x 3 = $36

2¢ x 3 = 6¢ or $0.06

Cost is $36.00 - $0.06 = $35.94

11. to calculate Manitoba sales tax given the percent rate of provincial sales tax and the price of an article or articles.

Example:

S. Talker in Brandon purchased a filing cabinet that cost $150. The Manitoba provincial sales tax is 5%. What is the tax that she will have to pay on this filing cabinet?

Methods for finding sales tax include:

1) multiplying the price by 0.05
2) moving the decimal place in the price 2 places to the left, and dividing that result by 2
3) multiplying the price by 5 and dividing that result by 100
4) multiplying the price by 5% using a calculator

OBJECTIVES | **NOTES TO TEACHERS**

Students should realize that all provinces except Alberta levy a sales tax on most goods and services purchased by consumers and businesses. The percent rate of and the regulations for the sales tax vary from province to province. Information about the percent rate and regulations related to sales tax can be obtained from the Taxation Division of the Department of Finance.

Have students realize what part sales tax plays in their daily lives and how it influences the actual price of an article.

B. PRICING

The student should be able:

1. to round off numbers involving dollars and cents;

Rounding off is important for obtaining approximations and in making estimates. Students should be taught to round off by writing only the figures that are needed and by dropping the remaining figures that lie to the right. If the first figure to be dropped is 5 or more, add 1 to the preceding digit. If the first figure to be dropped is less than 5, leave the preceding digit unchanged.

Examples:

Round off to the places indicated:

1. Nearest cent

   $2.3875, $163.439, $2.002, $1.006

2. Nearest dollar

   $28.25, $1825.50, $72.49, $248.42

3. Nearest ten dollars

   $61, $16, $136, $508

4. Nearest hundred dollars

   $457, $721, $23, $740

OBJECTIVES

2. to calculate the the cost of a specific number of articles given the price for a quantity purchase.

C. MAKING CHANGE

The student should be able to make change given the amount offered in payment.

NOTES TO TEACHERS

Examples:

1) An advertisement in the paper read: "5 packages of facial tissue for $1.79." What is the cost price of one package?

2) Find the cost of each of the following groups of items:

a) 2 items at 3 for $1.10
b) 3 items at 5 for $0.29
c) 4 items at 5 for $2.49
d) 7 items at 4 for $25.00

Students should be given a change sheet to complete when beginning to study the concept of making change.

After students have used the form for two or three problems, they should progress to counting change orally. Students should count from the smallest to the largest denomination.

CHANGE SHEET

Cost of Item	Amount Received	$.01	$.05	$.10	$.25	$1	$2	$5	$10	$20

Examples:

1) The price of an item is $2.79 and $5.00 is received from the customer. The student could fill in the above change sheet as follows: cost of item, $2.79; amount received, $5.00; and the amount of change to be given: 1, one-cent piece; 2, ten-cent pieces; and 1, two-dollar bill OR 1, one-cent piece; 2, ten-cent pieces; and 2, one-dollar bills.

2) The customer purchases an item priced at $38.02 and gives the cashier $40.02. Fill in the change sheet.

OBJECTIVES

NOTES TO TEACHERS

To help students understand the concept of making change, discuss the "equal trade process." For example, a customer gives the store clerk a $20 bill as trade for a $15.98 garment plus $4.02 change.

Example:

Louise White is a checker at the Busy Supermarket. How should Louise count the change due the following customers:

1) Customer A's total purchases are $17.88. He gives Louise two $10 bills.
2) Customer B's total purchases are $8.16. She gives Louise a twenty-dollar bill.

Op.III. DISCOUNT CALCULATIONS (1 module)

Manufacturers and wholesalers prepare catalogues and price lists for their goods, citing what retailers should charge their customers. From these catalogue list or retail prices, retailers themselves are entitled to a "trade discount," so named since granted only to those who retail merchandise. The trade discount is large enough to allow the retailer to make a net profit after paying cost and selling expenses. Retailers receive from manufacturers and wholesalers confidential lists known as trade discount sheets which give the percent rate of discount allowed for items in the catalogue and/or price lists. Therefore, a manufacturer can use the same catalogue for all customers (wholesale and retail), some of whom get larger discounts than others. This practice assures manufacturers that their products will retail at or around the list prices they set, since adjustments to fluctuating markets can be made by changing trade discount rates. Occasionally, however, additional discounts are made from the list price to meet competition or to secure new business.

Generally, since the buyer is entitled to the trade discount whether or not cash is paid, trade discounts are shown as deductions on an invoice, the goods being priced at the net amount after the discount has been taken off.

To encourage early payment for an invoice, sellers may allow cash discounts to purchasers. Although sellers lose the discount cost, they gain by receiving cash that helps them to take advantage of cash discounts offered by wholesalers and/or manufacturers, and by reducing costs associated with payment collection and losses due to bad debts.

OBJECTIVES	NOTES TO TEACHERS
A. SINGLE TRADE DISCOUNT The student should be able:	
1. to demonstrate an understanding of the meaning of these words: list price, retail price, retailer, manufacturer, wholesaler, trade discount, rate of trade discount, invoice price and net price;	Invoice price may not be the same as the list price. List price is written in the catalogue. Invoice price is what the manufacturer or wholesaler writes on the invoice.
2. to calculate the trade discount given the list price and the trade discount rate;	Formula: Trade Discount = List Price x Discount Rate Example: A suitcase lists at $135.00. A trade discount of 25% is offered. What is the amount of trade discount?

OBJECTIVES	NOTES TO TEACHERS
3. to find the net price given the rate of trade discount and the list price;	Formula: Net Price = List Price - Trade Discount Example: A car lists at $5800. A trade discount of 16% is offered. (a) What is the trade discount? (b) What is the net price?
4. to calculate the rate of trade discount given the amount of trade discount and the list price;	Formula: Rate of Trade Discount = Discount ÷ List Price Example: A lamp lists at $75 and sells for $60. What is the amount and rate of trade discount? (Note: Base discount rate on list price.)
5. to calculate the list price given the rate and amount of trade discount.	Formula: List Price = Trade Discount ÷ Trade Discount Rate Example: A trade discount of $200 is offered. The rate of trade discount is 10%. What is the list price? Students could use a microcomputer with a trade discount program as reinforcement for this section.
B. SERIES OF TRADE DISCOUNTS The student should be able:	
1. to explain the meaning of the term series of discounts or discount series;	When two or more trade discounts are offered on the same article, such discounts are known as a series of discounts or a discount series. The first discount applies to the list price; the second discount applies to the amount remaining after the first has been deducted; the third discount applies to the amount remaining after the first two discounts have been deducted, etc. To ensure accuracy in the cents column, have students carry the intermediate results correct to three decimal places. Round off the final result to the nearest cent.
2. to calculate the net price given the list price and a series of trade discount rates;	Examples: 1) An article lists at $1500. Trade discounts of 20% and 10% are offered. What is the net price? 2) An article lists at $1620. Trade discounts of 25% and 20% are offered. What is the net price? What is the amount of trade discount?

3. to find the amount of trade discount given the list price and a series of trade discount rates;

4. to find a single discount rate equivalent to a series of trade discounts;

Example:

Find the single discount rate equivalent to 20%, 10% and 5%. (This method is most efficient if several calculations involving the same percent rates are to be completed.)

Solution:

Step 1

List price	100.0%
Less: 1st discount	20.0%
First remainder	80.0%
Less: 2nd discount	8.0%
Second remainder	72.0%
Less: 3rd discount	3.6%
Third remainder or net price rate	68.4%

Step 2

List price	100.0%
Less: net price rate	68.4%
Single discount	31.1%

Note: When students are required to find a single discount equivalent to a series of trade discounts, they should apply the single discount equivalent as in the following example.

Example:

An article lists at $1500. Trade discounts of 20%, 10% and 5% are offered. What is the net price?

Solution:

Net price = List Price x Net Price Rate

Net price = $1500.00 x 68.4%

= $1026.00

OBJECTIVES

5. to find the net price, using a discount table given a series of trade discounts and the list price.

NOTES TO TEACHERS

An example of a discount table is:

Percent	5	7½	10	15	20	25	30
2	0.931	0.9065	0.822	0.833	0.784	0.784	0.686
2½	0.0263	0.9019	0.8775	0.8288	0.78	0.7313	0.6825
5	0.9025	0.8788	0.855	0.8075	0.76	0.7125	0.665
5 & 2½	0.8799	0.8568	0.8336	0.7873	0.741	0.6947	0.6484
7½	0.8788	0.8556	0.8325	0.7863	0.74	0.6938	0.6475
7½ & 5	0.8348	0.8128	0.7909	0.7469	0.703	0.6591	0.6151
10	0.855	0.8325	0.81	0.765	0.72	0.675	0.63

Example:

An article lists at $800. The trade discounts offered are 15%, 5% and 2½%. Using the discount table provided above, find the amount of trade discount and then calculate net price.

Solution:

Line up the 15% column with the row containing 5% and 2½%. The single discount equivalent for net price rate is 0.7873. Thus:

Net price = $800.00 x 0.7873 = $629.84

Amount of trade discount is $800 - 629.84 = $170.16

C. CASH DISCOUNT, CASH PRICE, AND INVOICES

The student should be able:

1. to distinguish between trade discounts and cash discounts;

If an invoice is paid within the discount period, the amount paid is the invoice price less the cash discount. This amount is designated as the net price/cash price. The percent indicating the rate of cash discount is always based on the invoice price. Thus, the invoice price is multiplied by the rate of the cash discount to find the amount of the cash discount.

OBJECTIVES

2. to explain the meaning of and the notation for "terms of sale";

3. to find the last date of payment for cash discount eligibility given the date of the invoice and the terms of sale;

NOTES TO TEACHERS

Invoice Price x Rate of Cash Discount = Cash Discount.

The cash discount is a deduction from the invoice price; therefore, it is subtracted from the invoice price to find the net price/cash price.

Invoice Price - Cash Discount = Cash Price.

The following is a list of typical terms of payment which might be offered.

n/30	The invoice must be paid within 30 days of its date. No cash discount is offered for prompt payment.
3/10, n/30	A 3% cash discount is offered if the bill is paid within 10 days of the date of the invoice. If the discount terms are not taken advantage of, the full amount must be paid within 30 days of the date of the invoice.
n/EOM	The abbreviation EOM stands for end of month. The bill must be paid by the last day of the month of the invoice. No cash discount is offered for prompt payment.
2/10 EOM	Regardless of the date of the invoice, 2% may be deducted if it is paid within 10 days of the last day of the month of the invoice. (Note: If an invoice is dated on or after the 26th of the month, a month's extension of time may be allowed.)

For simplicity, teachers could allow students to use a maturity table similar to the following:

MATURITY TABLE

DAY OF THE MONTH

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
JAN.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
FEB.	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59			
MAR.	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
APRIL	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	
MAY	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151
JUNE	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	
JULY	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212
AUG.	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243
SEPT.	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	
OCT.	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304
NOV.	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	
DEC.	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365

OBJECTIVES

NOTES TO TEACHERS

Example:

The date of an invoice is January 17. The terms of sale are 3/15, 2/20, n/30. Use the maturity table to determine the last date on which this invoice must be paid to take advantage of:

1) the 3% cash discount?
2) the 2% cash discount?

Solution:

1) January 17 + 15 days = February 1. (17 + 15 = 32)
2) January 17 + 20 days = February 6. (17 + 20 = 37)

4. to find the cash price given the list price, terms of sale, date of invoice and payment date;

Example:

An article lists for $2400. The invoice is dated October 28. The terms of sale are 5/10, n/30. What is the cash price if the invoice is paid on:

1) November 7?
2) November 9?

Solution:

1) Use the maturity date table to calculate the number of days involved:

Number of days from November 7 to October 28 (311 -301) is 10 days
List price = $2400.00
Less: Cash discount ($2400 x 5%) = 120.00
Cash price $2280.00

2) Number of days from November 9 to October 28 (313-301) is 12 days
List price = $2400.00
Less: Cash discount ($2400 x 0%) = 00.00
Cash price $2400.00
Note that since the invoice is not paid within 10 days, no cash discount is given.

5. to calculate the total cost given the list price, trade discount, date of invoice, terms of sale, date of payment and transportation charges;

The cost of transportation is one of the expenses of purchasing merchandise and having it delivered to a merchant's place of business. This expense consists mainly of parcel post, express, freight, and trucking charges. The transportation expense, which increases the cost of the goods purchased, is added to the invoice price, or to the net/cash price if a cash discount is taken.

Invoice Price [or Cash (Net) Price] + Transportation Expense = Total Cost.

OBJECTIVES

NOTES TO TEACHERS

Example:

An article lists at $1680. The terms of sale are 2/10, n/30. The date of the invoice is January 28 and payment is made on February 4. The transportation charges are $8.00. What is the total cost?

Solution:

List price	$1680.00
Less: Cash discount ($1680 x 2%)	33.60
Cash price	$1646.40
Plus: Transportation expense	8.00
Total cost	$1654.40

6. to explain f.o.b. terms and to calculate total charges involving f.o.b. amounts;

When goods are shipped by freight, the price quotation from the manufacturer or wholesaler generally specifies the shipping term as f.o.b. The letters f.o.b. stand for "free on board." When the seller agrees to pay the transportation charges, the shipping terms are designated f.o.b. destination. For example, the Brown Manufacturing Company, Winnipeg, quotes to L. A. Hill, Regina, a price of $150, f.o.b. Regina. This means the Brown Company will pay the freight charges from Winnipeg to the Regina freight depot.

When the buyer agrees to pay the transportation charges, the shipping terms are designated f.o.b. shipping point. For example, if the Brown Manufacturing Company, Winnipeg, quoted L. A. Hill, Regina, a price of $150, f.o.b. Winnipeg, L. A. Hill would have to pay the freight charges.

Example:

An article lists at $200. The freight charges are $9.00.

1) An article is shipped f.o.b. your town. What is the total cost of the article to you?
2) The article is shipped f.o.b. Winnipeg. What is the total cost to a buyer in Dauphin?

Answers:

1) $200
2) $209

OBJECTIVES

7. to answer questions regarding a given invoice form;

NOTES TO TEACHERS

Example:

Study the invoice and answer the questions that follow.

INVOICE
THE BETTER BUILT FURNITURE COMPANY

Winnipeg, Manitoba
R3G 0T3

TO: Atlas Department Store
WINNIPEG, Manitoba
R4Z 3L9

TERMS: 3/10, n/30

DATE: May 4, 19__

QUANTITY	DESCRIPTION	UNIT PRICE	AMOUNT
3	Sofas No. 467	200.00	

Questions:

1. What is the total cost of the three sofas?
2. What are the terms of the invoice?
3. What is meant by these terms?
4. By what date must the Atlas Department Store pay this invoice in order to take advantage of the 3% cash discount rate?
5. What is the last date on which the invoice may be paid? (Use maturity table.)

OBJECTIVES

8. to calculate the total cost given an invoice form.

NOTES TO TEACHERS

Answers:

1. $600.00.
2. 3/10, n/30.
3. A 3% discount will be given if the bill is paid within 10 days of May 4th. Otherwise, the full amount must be paid within 30 days of May 4th.
4. May 14.
5. June 3.

Example:

Find the total cost to the buyer if the following invoice is paid on April 16. The transportation charges from Winnipeg to Ottawa are $27.00. The rates of trade discount are 20% and 10%. The goods are shipped from the wholesaler in Winnipeg to the retailer in Ottawa.

167 Buchanan Boulevard, WINNIPEG, Manitoba
CRAMER'S STATIONERY COMPANY Wholesale Office Supplies

Sold To: Leslie Palmer, Stationer
69 Market Street
Ottawa, Ontario
K3G 1S0

Date: April 10, 19__

F.O.B. Winnipeg

Terms: 2/10, n/30

QUANTITY	DESCRIPTION	UNIT PRICE	AMOUNT
24	Pen and Pencil Sets	1.10	26.40
12 pkg.	Notebook Fillers	1.25	15.00
36 bx.	Carbon Paper	1.50	54.00
16 bx.	File Folders	1.75	28.00
			123.40

Note: Students should be able to do calculations of this type using a single equivalent discount table.

OBJECTIVES	NOTES TO TEACHERS

Solution:

Net price ($123.40 x 0.72)	=$ 88.85
Less: Cash discount (88.85 x 2%)	= 1.78
Cash price	$ 87.07
Plus: Transportation charge	27.00
Total cost	$114.07

Op.IV. MERCHANDISE AND SERVICE BUSINESS CALCULATIONS (1 module)

This module contains concepts related to the following business operations: markon (markup), markdown and net income.

OBJECTIVES **NOTES TO TEACHERS**

A. MARKON (MARKUP) AND MARKDOWN

Markon (markup) and markdown are important concepts in establishing retail prices for merchandise which a business sells. To establish the price of goods to be sold, every retail business must consider the selling price of similar articles in other stores and the need to produce a profit.

The student should be able:

1. to demonstrate an understanding of the following terminology as applied to business: markon, selling price, and markdown;
2. to find the amount of markon and the selling price, given the percent rate of markon (based on the cost) and the cost;

Students must be aware that if a retailer is to pay the expenses of operating a business and still make a profit, the merchandise must be priced at a higher amount than the amount paid for it. The difference between the selling price of an article and the cost of an article is called markon.

Example:

If Mr. Merny decides to sell for $65 the jackets that he bought for $40, find the markon.

Solution:

Selling price:	$65.00
Less: Cash price	40.00
Markon (profit):	$25.00

Develop the formula:

COST + MARKON = SELLING PRICE

Students should realize that the following are other variations of this same formula:

Selling Price - Cost = Markon

Selling Price - Markon = Cost

Students should not be expected to know all variations of a formula but rather should work toward formula manipulation. Refer to C.IV Algebra.

OBJECTIVES

3. to calculate the percent rate of markon based on cost, given the cost and the selling price;

NOTES TO TEACHERS

Example:

An article costs $60. The percent rate of markon based on the cost is 15%. What is the amount of the markon and the selling price?

Solution:

Cost = $60.00
Plus: Markon ($60.00 x 15%) = 9.00
Selling price $69.00

Examples:

1) An article which costs $50 sells for $60. What is the markon and what is percent rate of markon based on cost?

Solution:

Selling price $60.00
Less: Cost 50.00
Markon 10.00
Rate of markon on cost ($10.00 ÷ 50.00) = 20%

2) Find the markon and the percent rate of markon based on cost in each of the following. (Compute the percent rate of markon to the nearest whole number.)

Item	Cost	Selling Price	Markon	% Rate of Markon Based on Cost
A	$ 74.00	$ 80.00	$______	______%
B	$ 17.50	$ 24.05	$______	______%
C	$ 37.50	$ 50.00	$______	______%
D	$137.20	$222.95	$______	______%

OBJECTIVES

4. to calculate the percent rate of markon based on the selling price;

NOTES TO TEACHERS

Some retail businesses prefer to state markon as a percent rate of the selling price rather than as a percent rate of the cost price. Using this method, they can state the markon rate at a lower percent. Example: Brown's Clothing buys an article for $50 and sells it for $60. Determine the percent rate of the markon based on the cost price.

Solution:

Selling price	$60.00
Less: Cost	50.00
Markon	$10.00

Rate of markon on selling price (10.00 ÷ 60.00) = 16.7% (17%, if rounded to the nearest whole number)

Examples:

1) An article costing $54.00 sells for $57.60. Calculate the markon and the percent rate of markon based on the selling price.

Solution:

Selling price	$57.60
Less: Cost	54.00
Markon	$ 3.60

Rate of markon on selling price (3.60 ÷ 57.60) = 6%

2) Find the markon and the percent rate of markon based on the selling price for each item in the following table. To the nearest whole number, compute the percent rate of markon based on the selling price for each item.

Item	Cost	Selling Price	Markon	% Rate of Markon Based on Selling Price
A	$ 37.50	$ 50.00	$______	______%
B	$ 50.00	$ 60.00	$______	______%
C	$ 54.00	$ 57.60	$______	______%
D	$129.50	$148.00	$______	______%

OBJECTIVES

5. to calculate the amount of markdown, given the original selling price and the reduced price (new selling price);

6. to calculate the percent rate of markdown, given the original selling price and the reduced price.

NOTES TO TEACHERS

Retail businesses often reduce prices on merchandise for the following reasons: to sell old merchandise that is not selling well, to clear out shop-worn articles, to meet the prices of competition, and to attract new customers through "sales." The reduction in the regular selling price is a markdown.

To find the markdown, subtract the reduced price (new selling price) from the original selling price.

ORIGINAL SELLING PRICE - REDUCED PRICE = MARKDOWN

Example:

An article originally priced at $50.00 is reduced to $40.00. What is the markdown?

Solution:

Original price	$ 50.00
Less: Reduced price	40.00
Markdown	$ 10.00

Teachers should emphasize that for inventory purposes retail businesses express the percent rate of markdown as a percent rate of the new reduced price rather than as a percent rate of the original price.

Examples:

1) An article originally priced at $50.00 is reduced to $40.00. What is the percent rate of markdown?

Solution:

Original price	$ 50.00
Less: Reduced price	40.00
Markdown	$ 10.00

Rate of markdown (10.00 ÷ 40.00) = 25%

2) Find both the markdown and the percent rate of markdown for each article: (Complete the percent rate of markdown to the nearest whole number.)

Item	Original Price	Reduced Price	Markdown	% Rate of Markdown
A	$ 60.00	$ 40.00	$_____	_____%
B	$ 9.00	$ 7.50	$_____	_____%
C	$ 70.00	$ 42.00	$_____	_____%
D	$ 17.50	$ 12.50	$_____	_____%

Note: Near the end of this chapter, students could use the microcomputer to review the concepts of markup and markdown.

B. INCOME STATEMENT

This section of the module introduces the student to the mathematics associated with preparing income/profit statements, to the concepts of revenues, expenses, and profits, and to elementary concepts related to formating income statements. The examples cited do not include the Cost of Goods Sold statement which is reserved for subsequent grade levels.

The student should be able:

1. to calculate total revenues given a variety of sources;

Teachers should provide a variety of examples to illustrate the concepts of revenue, costs and expenses, and profits. (Revenue is defined as the sum of all monies received by a firm or individual.)

Example 1:

The student council of a high school raises money through a number of activities. Revenues from these activities include the following:

a) Hot Dog Sale (330 @ 50¢) =$165.00
b) T-Shirt Sale (170 @ $5.00)= 850.00
c) Sock-Hop (200 @ 50¢) = 100.00
d) Drink Machine (250 @ 50¢) = 125.00
e) Student Fees (200 @ $2.00)= 400.00

Calculate the total revenue for the student council.

Example 2:

Over a month, a lawyer receives revenues from many sources:

a) Consultation Fees (12 @ $100.00)= $1200.00
b) Mortgage Drafts (9 @ $600.00) = 5400.00
c) Wills Drawn (6 @ $50.00) = 300.00
d) Property Sale Transactions (9 @ $500.00) = 4500.00
e) Transfers from other lawyers (6 @ $100.00) = 600.00

Calculate the total revenue for the lawyer.

OBJECTIVES

NOTES TO TEACHERS

Solution:

In equation form, this concept can be illustrated as follows:

TOTAL REVENUE = SALES + FEES
=(165+850+125)+(100+400)
=$1140+$500
=$1640.00

The teacher should explain the format of a statement in which revenues are classified.

In statement form, this concept can be illustrated as follows:

REVENUES

Sales		
Hot Dogs	$165.00	
T-Shirts	850.00	
Soft Drinks	125.00	
Total Sales		$1140.00
Fees		
Dance	100.00	
Student Fees	400.00	
Total Fees		500.00
Total Revenue		$1640.00

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Solution:

In equation form, this concept can be illustrated as follows:

TOTAL REVENUE = FEES + TRANSFERS
=(1200+5400+300+4500)+(600)
=$11 400+$600
=$12 000.00

In statement form, this concept can be illustrated as follows:

REVENUES

Fees		
Consultations	$1200.00	
Mortgage Drafts	5400.00	
Wills Drawn	300.00	
Property Sales Transactions	4500.00	
Total Fees		$11 400.00
Transfers from other lawyers		600.00
Total Revenue		$12 000.00

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OBJECTIVES

2. to calculate net revenue given sales returns and/or allowances;

NOTES TO TEACHERS

Note: The sources for total figures need not be disclosed when statements become more complex, as the following Example I indicates by showing the total sales as $1140.00, leaving the component parts undisclosed.

Example I:

Referring to the previous Example I, what would be the net revenue if: 5 T-shirts @ $5.00 each were returned because of defective decals, and 4 student fees @ $2.00 were refunded because of students transferring?

Solution:

In equation form, this concept can be illustrated as follows:

NET REVENUE = NET SALES + NET FEES
= (1140-25)+(500-8)
= $1115+$492
= $1607.00

Example 2:

Referring to the previous Example 2, what would be the net revenue if one disgruntled client received credit for $200.00 worth of consultation fees and $50.00 worth of transfers made on his behalf?

Solution:

In equation form, this concept can be illustrated as follows:

NET REVENUE = NET FEES + NET TRANSFERS
= (11 400-200)+(600-50)
= $11 200+$550
= $11 750.00

OBJECTIVES	NOTES TO TEACHERS

In statement form, this concept can be illustrated as follows:

NET REVENUE

Sales................	$1140.00	
Less: Sales Returns	25.00	
Net Sales....................		$1115.00
Fees.................	500.00	
Less Fees Refunded	8.00	
Net Fees.....................		492.00
Net Revenue....................		$1607.00

In statement form, this concept can be illustrated as follows:

NET REVENUE

Fees..............	$11 400.00	
Less: Fees Refunded..	200.00	
Net Fees..................		$11 200.00
Transfers from other lawyers...	600.00	
Less: Transfers Credited...	50.00	
Net transfers.............		550.00
Net Revenue.................		$11 750.00

3. to calculate total expenses given a variety of expenditures;

In this course, expenses are defined as costs to a firm or individual.

Note: The teacher should discuss the merits of disclosing a zero-balance account, versus leaving it unreported, since disclosure indicates that it has not been overlooked.

Example 1:

The student council incurred the following costs in promoting its activities:

a) weiners 330 @ 25¢
b) buns 330 @ 10¢
c) relish & mustard 330 @ 02¢
d) T-Shirts 170 @ $3.00
e) janitor for dance @ $35.00
f) soft drinks 250 @ 40¢

Calculate the total expenses.

Example 2:

A lawyer incurred the following expenses during a month:

a) rent	$ 500.00
b) secretary	1200.00
c) utilities	400.00
d) travel	1500.00
e) disbursements to other agencies	$400.00
f) miscellaneous	500.00

Calculate the total expenses.

OBJECTIVES

NOTES TO TEACHERS

Solution:

In equation form, this concept can be illustrated as follows:

TOTAL EXPENSES = HOT DOG EXPENSES + T-SHIRT COSTS + JANITOR + SOFT DRINKS + STUDENT CARDS

= 122.10+510.00+35.00+100.00+n/c

= $767.10

In statement form, this concept can be illustrated as follows:

TOTAL EXPENSES

Hot Dog Sale		
Weiners (330x25¢) =	$82.50	
Buns (330x10¢) =	33.00	
Relish & Mustard (330x2¢) =	6.60	
Total Hot Dog Expenses		$122.10
T-Shirt Sale		
T-Shirts (170x$3.00)		510.00
Sock-Hop		
Janitorial Service		35.00
Drink Machine		
Soft Drinks (250x40¢)		100.00
Student Cards		n/c
Total Expenses		$767.10

Solution:

In equation form, this concept can be illustrated as follows:

TOTAL EXPENSES = RENT + WAGES + UTILITIES + TRAVEL + DISBURSEMENTS + MISCELLANEOUS

= 500+1200+400+1500+400+500

= $4500.00

In statement form, this concept can be illustrated as follows:

TOTAL EXPENSES

Rent	$ 500.00
Wages	1200.00
Utilities	400.00
Travel	1500.00
Disbursements	400.00
Miscellaneous	500.00
Total Expenses	$4500.00

OBJECTIVES

4. to explain the difference between revenues, expenses, and profits;

5. to prepare an income statement given revenues, sales returns and allowances, and expenses.

NOTES TO TEACHERS

Note: The term net income could be used interchangeably with net profit/loss.

By formula, profit may be determined as follows:

Net Income = Net Revenue - Total Expenses

Students should be able to prepare simple income statements by using the data provided in the foregoing examples.

Example 1:

STUDENT COUNCIL
INCOME STATEMENT

for the year ended June 30, 19__.

REVENUES		
Sales...............	$1140.00	
Less: Sales Returns	25.00	
Net Sales....................		$1115.00
Fees................	$ 500.00	
Less: Fees Refunded	8.00	
Net Fees....................		492.00
NET REVENUES..................		$1607.00
EXPENSES		
Hot Dog Sale........	$ 122.10	
T-Shirt Sale........	510.00	
Sock-Hop............	35.00	
Drink Machine.......	100.00	
Student Fees........	n/c	
TOTAL EXPENSES................		$ 767.10
NET INCOME....................		$ 839.90

Example 2:

LAWYER PROP.
INCOME STATEMENT

for the period ended June 20, 19__.

REVENUES		
Fees............	$11 400.00	
Less: Fees Refunded	200.00	
Net Fees..................		$11 200.00
Transfers from other lawyers	600.00	
Less: Transfers Credited	50.00	
Net Transfers...............		$ 5 000.00
NET REVENUES................		$11 750.00
EXPENSES		
Rent...........	500.00	
Wages..........	1 200.00	
Utilities......	400.00	
Travel.........	1 500.00	
Disbursements....	400.00	
Miscellaneous....	500.00	
TOTAL EXPENSES..............		$ 4 500.00
NET INCOME..................		$ 7 250.00

VOCATIONAL MATHEMATICS

Op.V. MEASUREMENT AND ESTIMATION (1 module)

Teachers should check with Industrial Arts or Science teachers to determine the availability of measuring devices in their schools. Teaching models (plastic) are available from some supply houses.

OBJECTIVES

A. MEASURING INSTRUMENTS

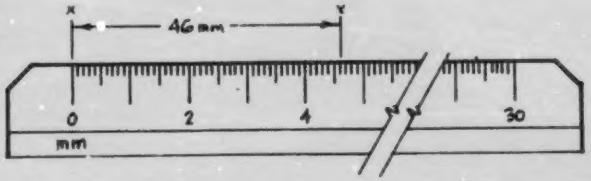
The student should be able:

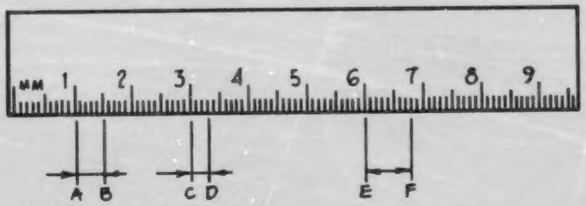
1. to read ruler scales to the nearest millimetre;

NOTES TO TEACHERS

Note: Teachers are reminded that size reduction of these diagrams may cause inaccuracies in reading scales.

Example:





From the above diagram, determine and record the indicated lengths.

Answers: 1) AB = 5 mm 2) CD = 3 mm 3) EF = 8 mm

Note: The measurement should be estimated to the nearest millimetre.

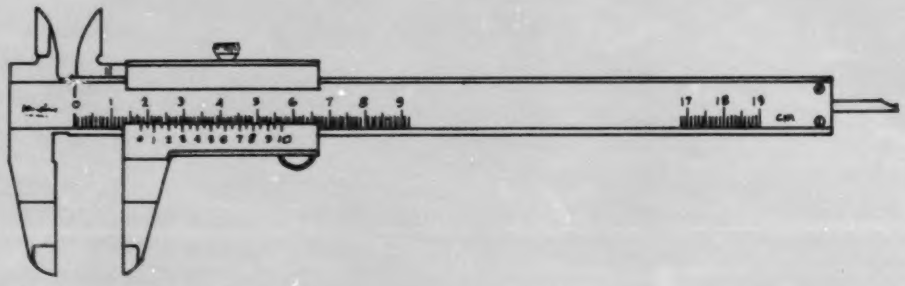
OBJECTIVES

2. to read metric vernier calipers;

NOTES TO TEACHERS

Major fixed scale divisions are in centimetres with each millimetre subdivision marked. Each millimetre is further divided into 10 parts on the sliding vernier scale (0.1 mm); each of the vernier divisions is divided into 2 parts (½ x 0.1 = 0.05 mm). Read the fixed scale mm division to the immediate left of the vernier 0. Then scan the vernier to determine the vernier mark which lines up with a main scale mark. This will either be a 0.1 mm or a 0.05 mm mark. Thus, readings to the nearest .05 mm may be made by combining the two scale readings.

Example:



In the above example:

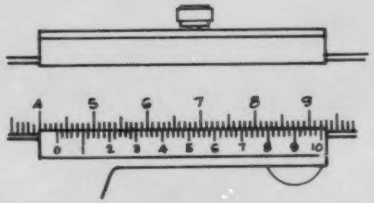
Main scale division	18 mm	OR	1.8 cm
vernier division	.25 mm		.025 cm
Reading	18.25 mm		1.825 cm

Note: Vernier calipers vary somewhat in design and it is suggested that teachers consult the manufacturer's literature for reading instructions on specific makes.

OBJECTIVES

NOTES TO TEACHERS

Example:


4
5
6
7
8
9
0
1
2
3
4
5
6
7
8
9
10

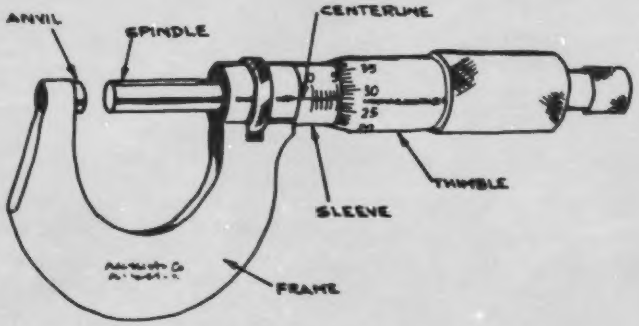
Here each vernier division is divided into 5 parts, each worth 0.2 of the major vernier divisions.

Read:		
	Main scale	43 mm
	Vernier scale	.18 mm
	Reading	43.18 mm

Note that the fourth division between 1 and 2 is lined up, and thus the reading is .18.

3. to read metric micrometers.

Micrometers are used in machine shops where the cutting and shaping of parts to a specific size take place. Mechanics use micrometers for checking wear and measuring cylinders, brake drums, brake rotors, and bearings.


ANVIL
SPINDLE
CENTERLINE
35
30
25
0
5
SLEEVE
THIMBLE
FRAME

OBJECTIVES	NOTES TO TEACHERS

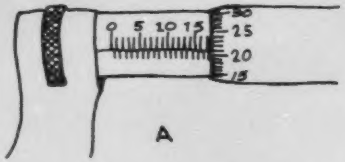
The thimble screw has 20 threads per cm. Therefore, 1 turn = 1/20 cm or 0.5 mm. Two turns will move the spindle 1 mm and the leading edge of the thimble, along the main sleeve scale, the same distance. Most sleeves are graduated in either 1 or 0.5 mm divisions. The vernier on the thimble has 50 divisions and two turns gives 100 divisions for each mm of travel on the spindle. Readings to the nearest 0.01 mm can therefore be made.

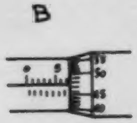
Example:

Read the last sleeve mm graduation mark exposed to the immediate left of the thimble's leading edge. On the thimble read the mark that most nearly lines up with the center-line.

Read:		
	Sleeve	5. mm
	Thimble	.28 mm
	Reading	5.28 mm

Example:





Read:

A.	Sleeve:	17.5 mm	B.	Sleeve:	7. mm
	Thimble:	.21 mm		Thimble:	.425 mm (est.)
	Reading	17.71 mm		Reading:	7.475 mm = 7.48 mm

OBJECTIVES

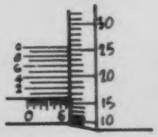
B. READING DIAGRAMS

The student should be able:

1. to read measurements from diagrams;

NOTES TO TEACHERS

Example:

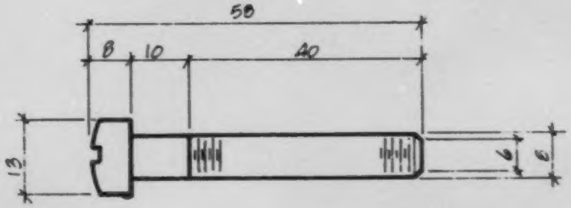


Micrometers may also have a second vernier on the sleeve which subdivides the thimble vernier.

Read:		
	Sleeve:	6. mm
	Thimble:	.16 mm
	Sleeve Vernier:	.003 mm
	Reading	6.163 mm

Note: Consult resource material for further explanation and manufacturer's literature for information on specific models.

Example:



Questions: What is the (1) bolt head diameter?
(2) bolt head thickness?
(3) bolt length?

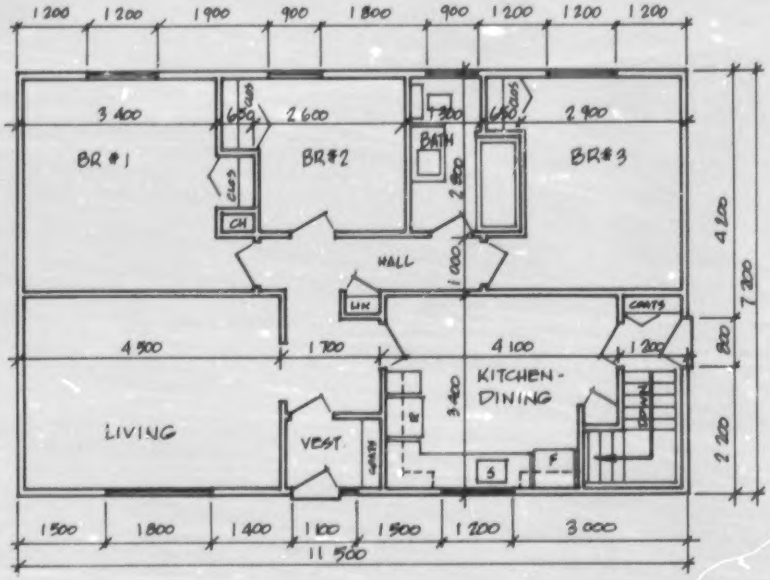
Answers:
(1) 13 mm
(2) 8 mm
(3) 58 mm

OBJECTIVES

2. to read measurements from floor plans;

NOTES TO TEACHERS

Example:



1 200
1 200
1 900
900
1 800
900
1 200
1 200
1 200
3 400
650
2 600
1 300
650
2 900
BR #1
BR #2
BATH
2 800
BR #3
CLOS
CLOS
CH
HALL
1 000
LIN
COATS
4 200
7 200
4 500
1 700
4 100
1 200
800
KITCHEN-DINING
3 400
LIVING
VEST
DOWN
2 200
S
F
1 500
1 800
1 400
1 100
1 500
1 200
3 000
11 500

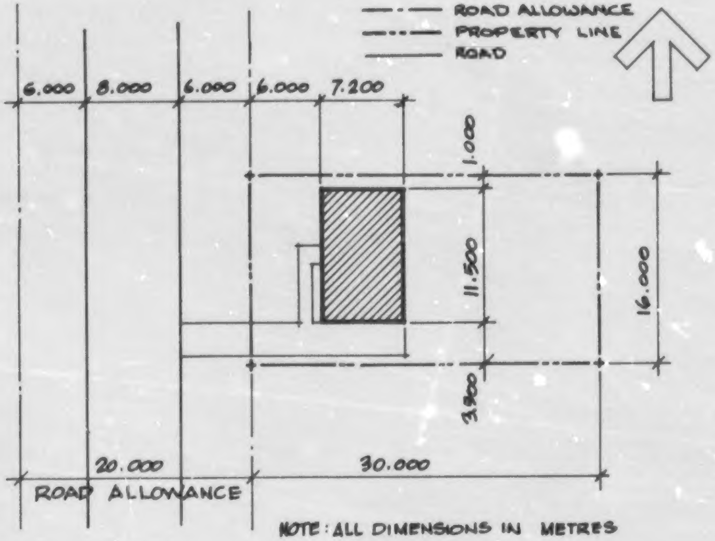
FLOOR PLAN Scale: 1:100
Note: All dimensions are in millimetres unless noted otherwise.

Questions: What are the dimensions of (1) the living room?
(2) bedroom #3?

Answers: (1) 4500 mm x 3400 mm
(2) 3800 mm x 2900 mm

Floor plans may be obtained from newspapers, magazines, books of house plans, local builders and building supply stores.

OBJECTIVES	NOTES TO TEACHERS
3. to read measurements from site plans.	Example:


ROAD ALLOWANCE
PROPERTY LINE
ROAD
6.000
8.000
6.000
6.000
7.200
1.000
11.500
16.000
3.500
20.000
30.000
ROAD ALLOWANCE
NOTE: ALL DIMENSIONS IN METRES

Questions: How far is the house set back from:

(1) the front property line?
(2) the north property line?
(3) the road?

Answers: (1) 6.000 m
(2) 1.000 m
(3) 12.000 m

353

354

OBJECTIVES

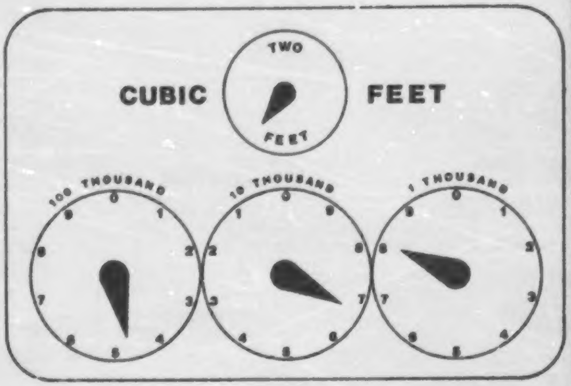
C. READING METERS

The student should be able:

1. to read natural gas and hydro meters;

NOTES TO TEACHERS

Example:

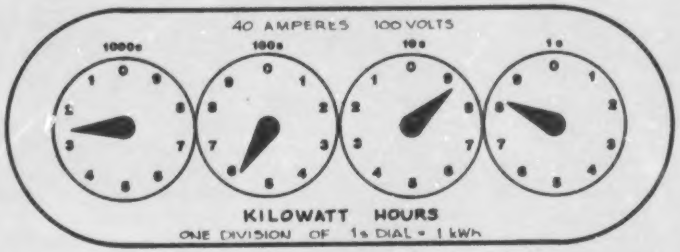

CUBIC
TWO
FEET
FEET
100 THOUSAND
10 THOUSAND
1 THOUSAND

Question: What is the reading of the above gas meter?

Answer: 468 thousand or 468 000.

OBJECTIVES | **NOTES TO TEACHERS**

Example:


40 AMPERES
100 VOLTS
1000s
100s
10s
1s
KILOWATT HOURS
ONE DIVISION OF 1s DIAL = 1 kWh

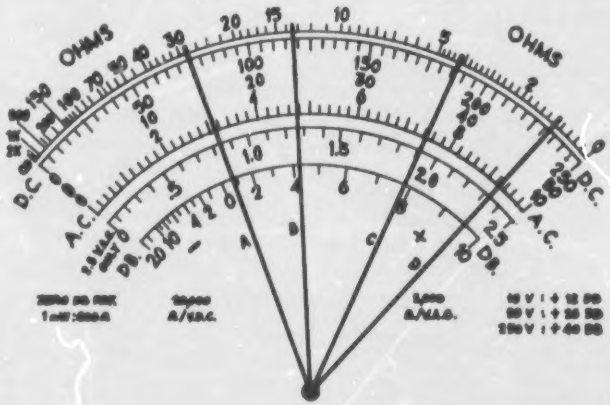
Question: What is the reading of the above hydro meter?

Answer: 2588 kwh

2. to read voltmeters, ohmmeters, and/or ammeters;

Example:

In this diagram a single needle is shown in four different positions (A, B, C, D).


OHMS
OHMS
D.C.
D.C.
A.C.
A.C.
DB.
DB.
A
B
C
D
1.0
1.5
2.0
2.5

OBJECTIVES

3. to read common graduated scales.

NOTES TO TEACHERS

The volt-ohmmeter (VOM) dial shown has several scales and ranges which are chosen by a selection switch on the meter case.

The AC scale is read radially outward in the ranges 0-10, 0-50, 0-250V. The DC scale is read radially inward in the ranges 0-250, 0-50, 0-10V.

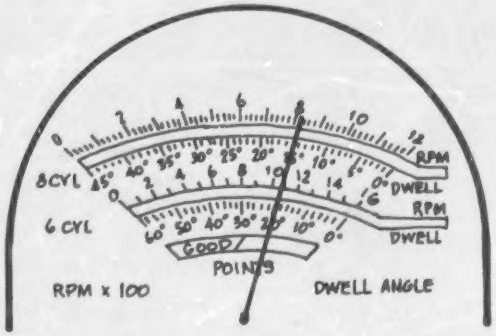
The outer scale is ohms, (Ω).

The scale ranges usually can be modified by switch settings that vary by a power of 10. Thus, 0-250VDC may be 0-25V, 0-2.5V.

Needle position A:	0.75 dB	
	0.85V AC	
	3.1 V AC	3V DC
	15.5 V AC	15V DC
	77.5 V AC	75V DC
	28 Ω	

Note: The dynamic properties of the meter movement are not precisely the same for AC as for DC. The AC scale is not linear which leads to an offset of the AC and DC graduations and thus requires two scales. The effect is most marked at the lower end of the AC scale.

Example:


0
2
4
6
8
10
12
RPM
45° 40° 35° 30° 25° 20° 15° 10° 5° 0°
8CYL
0 2 4 6 8 10 12 14 16
DWELL
RPM
6 CYL
60° 50° 40° 30° 20° 10° 0°
DWELL
GOOD
POINTS
RPM x 100
DWELL ANGLE

Question: What is the rpm and dwell of the 8-cylinder engine?

Solution: 8.2 x 100 = 820 rpm; 15°

For the 6-cylinder engine, dwell = 20°, rpm = 1100.

Common graduated scales could include thermometers, weigh scales, pressure gauges, tachometers, and speedometers.

OBJECTIVES

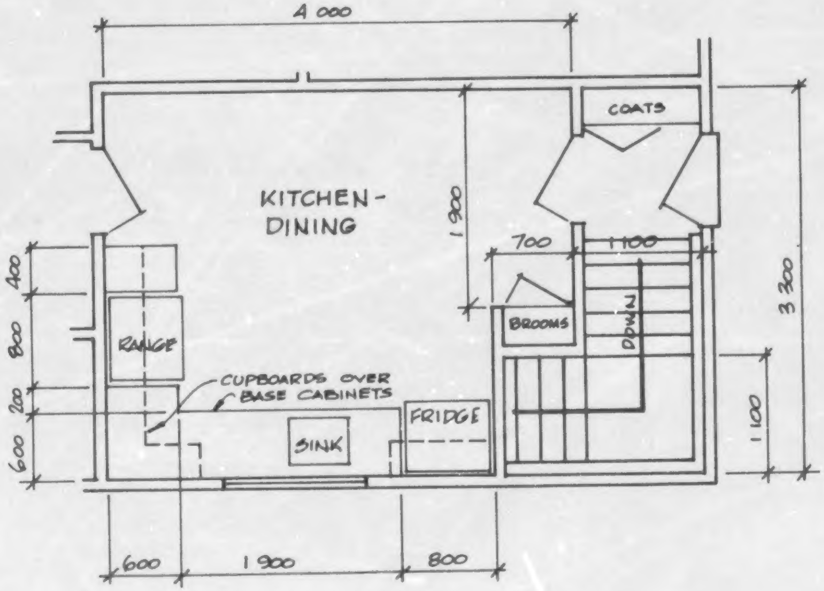
D. COST CALCULATIONS

The student should be able:

1. to determine area and to calculate the total cost given the cost per unit area;

NOTES TO TEACHERS

Example:


4 000
COATS
KITCHEN-DINING
1 900
700
1100
3 300
BROOMS
DOWN
400
800
200
600
RANGE
CUPBOARDS OVER BASE CABINETS
FRIDGE
SINK
1 100
600
1 900
800

DETAILED PLAN SCALE: 1:50

Note: All dimensions are in millimetres, unless otherwise noted.

Question: If vinyl floor covering costs $9.95 per square metre, how much would it cost to cover the kitchen-dining area?

Solution:

Several solutions are possible, depending on how the kitchen measurements are taken and on the width of the flooring.

OBJECTIVES

NOTES TO TEACHERS

2. to determine volume and to calculate the total cost given the cost per unit volume;

Example:

A driveway 3 m wide by 10 m long by 15 cm is to be paved. If concrete costs \$70/m^3, what is the total cost of the concrete?

Solution:

$$\text{Total volume} = 3 \times 10 \times \frac{15}{100}$$

$$= 3 \times 10 \times 0.15$$

$$= 4.5 \text{ m}^3$$

$$\text{Total concrete cost} = 4.5 \times \$70$$

$$= \$315$$

3. to solve problems involving time and money calculations;

Example 1:

A mechanic is paid \$8.25 an hour. How much time should he take to complete a job if the allowance for labor is \$78.37?

Solution: $78.37 \div 8.25 = 9.5$ hours.

Example 2:

A machinist's time card shows he started on a job at 8:28 a.m. and finished it at 3:12 p.m. Allowing an hour for lunch and two 15-minute rest breaks, how much labor is charged the customer at \$22 an hour? How much does the machinist make if the shop pays him \$14.48 per hour?

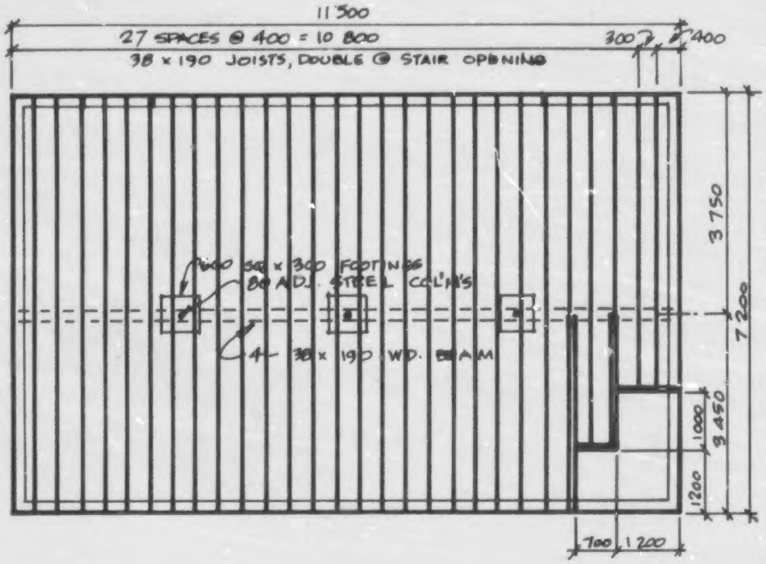
Answer: \$115.28; \$75.87.

OBJECTIVES

4. to estimate the quantity of materials needed for particular jobs.

NOTES TO TEACHERS

Example:


11 500
27 SPACES @ 400 = 10 800
300
400
38 x 190 JOISTS, DOUBLE @ STAIR OPENING
600 x 600 x 300 FOOTINGS
80 ADJ. STEEL COL'M'S
4 - 38 x 190 WD. BEAM
3 750
7 200
3 450
1000
1200
700
1 200

-JOIST PLAN - SCALE: 1:100

Note: Metric plywood is now available in sheets 1200 x 2400 mm. The old 4' x 8' sheet has a nominal dimension of 1224 x 2496 mm. Change mm to m before calculating area.

Question: Estimate the number of sheets of metric plywood which would be needed for the sub-floor shown in the diagram. (The opening for the stairwell is not to be covered.)

Solution:

Total area = (11.5 x 7.2) = 82.8

1 sheet = 1.2 x 2.4 = 2.88 m^2

Number of sheets = $\frac{82.8}{2.88}$

= 28.75 sheets, which means that approximately 30 sheets appear to be necessary.

OBJECTIVES

E. MASS CALCULATIONS

The student should be able:

1. to determine the value of the unknown, given two of the three quantities: mass per unit area, total mass, and total area;

2. to determine the value of the unknown, given two of the three quantities; mass per unit volume, total mass, and total volume.

NOTES TO TEACHERS

Example:

A footing 20 m^2 in area supports a mass of 35 000 kg. How many kilograms are supported per square metre? By 2 m^2?

Solution:

$$\frac{35\ 000}{20} = 1750 \text{ kg/m}^2;\ \times 2 = 3500 \text{ kg/m}^2$$

Example:

A certain type of concrete has a mass per unit volume of 2100 kg/m^3. Determine the mass of a concrete wall 5 m by 3 m by 0.25 m.

Solution:

$$\text{Total volume} = 5 \times 3 \times 0.25$$

$$= 3.75 \text{ m}^3$$

$$\text{Total mass} = \text{Volume} \times \frac{\text{mass}}{\text{unit volume}}$$

$$= 3.75 \times 2100$$

$$= 7875 \text{ kg}$$

$$= \underline{7.875 \text{ tonnes}}$$

Op.VI. RATIO AND PROPORTION (1 module)

OBJECTIVES

A. RATIO

The student should be able:

1. to explain the meaning of the word "ratio" and to illustrate the standard ways of writing ratios;

2. to reduce ratios to lowest terms;

NOTES TO TEACHERS

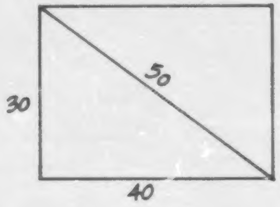
A comparison (quotient) of two quantities is a ratio.

Ways to express a ratio include:

1) the ratio of 3 to 5
2) 3/5
3) 3:5
4) 3 to 5

Example:

Determine the ratio of the diagonal to the short side in the following diagram.



$$\frac{\text{diagonal}}{\text{short side}} = \frac{50}{30} = \frac{5}{3}$$

Example:

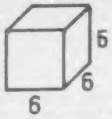


Fig. A

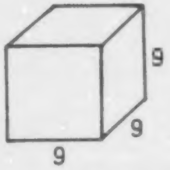


Fig. B

1) First degree: $\frac{\text{side A}}{\text{side B}} = \frac{6}{9} = \frac{2}{3}$
(linear)

OBJECTIVES | NOTES TO TEACHERS

2) Second degree: (Area)

$$\frac{\text{Face area A}}{\text{Face area B}} = \frac{(6)^2}{(9)^2} = \frac{36}{81} = \frac{4}{9}$$

3) Third degree: (Volume)

$$\frac{\text{Volume A}}{\text{Volume B}} = \frac{(6)^3}{(9)^3} = \frac{216}{729} = \frac{8}{27}$$

Note: Ratios involving circles, spheres, cylinders, cones, and frustums may be used as enrichment material for students who are capable, but this material should, in general, be left for development in grade 11.

3. to reduce ratios so that a numerator or denominator of one (1) results;

Example:

$$\frac{5}{3} = \frac{5 \div 3}{3 \div 3} = \frac{1.6\overline{6}}{1}$$

Divide both the numerator or denominator by either the numerator or denominator as required.

Note: Gear ratios are often expressed in this manner.

4. to express the rate of automobile fuel consumption in L/100 km.

Example:

An automobile used 28 L of fuel to travel a distance of 350 kilometres. Find the fuel economy.

$$\text{Fuel economy} = \frac{\text{fuel consumed}}{\text{distance travelled}} = \frac{28}{350} = \frac{28 \div 3.5}{350 \div 3.5} = \frac{8\ \text{L}}{100\ \text{km}}$$

OR 8 L/100 km.

OBJECTIVES

B. APPLICATION OF RATIOS

The student should be able to apply the concept of ratio to solving problems that occur in business, trade, or industry.

NOTES TO TEACHERS

Teachers are advised to select applications based on the interests and aptitudes of their students. Examples may be chosen from business and consumer areas and may include profit return on capital investment, outboard motor oil and fuel mixtures, pitch ratio of a roof, scale drawings, teeth ratio in gears, rear-axle ratios, etc.

Example 1: Scale Drawings

The scale to which a drawing is constructed is always taken as the ratio of the drawing size to the real object size.

$$\text{Scale} = \frac{\text{Drawing Size}}{\text{Object Size}}$$

A scale of 1:3 means the object is three times larger than the drawing. A scale of 4:1 would mean the drawing is four times larger than the object, or the object is one-quarter the size of the drawing.

Example:

A student desk has a top that is 60 cm by 45 cm. If a scale drawing of the desk top is 12 cm by 9 cm, determine the scale of the drawing.

Solution:

Since the scale equals the drawing size divided by the object size, the ratio is set up so that 12 cm is divided by 60 cm, or 9 cm by 45 cm.

$$\text{Scale} = \frac{12\text{ cm}}{60\text{ cm}} \quad \text{OR} \quad \text{scale} = \frac{9\text{ cm}}{45\text{ cm}}$$

Reducing to lowest terms gives a scale of $\frac{1}{5}$.

Example 2: Teeth Ratio in Gears

A gear is a cogged wheel with teeth cut around the outside or circumference. It is fastened to a shaft or axle at its center, and the teeth are designed to mesh with other gears for the transmission of rotary motion.

Spur gears - gears that have teeth projecting radially outward from the center, in the plane of the gear body. They are used for transmitting power between parallel shafts. (See Example: Meshing Gear)

Bevel gears - gears that have teeth cut at an angle to the plane of the gear and are used to transmit power between shafts that have an angle (usually 90°) between them. (See Example: Rear-Axle Ratio)

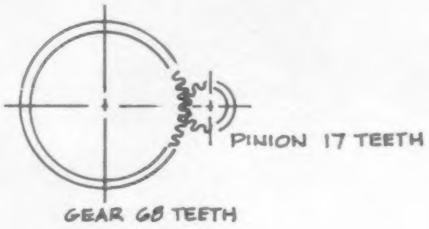
Crown gears - gears that have the teeth cut parallel to the plane of the gear and project at right angles to it.

Ring gear - the large rear axle gear in the differential gear assembly of an automobile.

Pinion gear - the small driveshaft gear that is in mesh with the large ring gear in a differential gear assembly.

Teeth ratio is the ratio of the number of teeth in one gear to the number of teeth in a gear meshing with it.

Example: Meshing Gear



Question: From this diagram determine the teeth ratio of 1) gear to pinion, 2) pinion to gear.

Solution: 1) Teeth ratio $= \frac{\text{gear}}{\text{pinion}} = \frac{68}{17} = \frac{4}{1}$ 2) Teeth ratio $= \frac{\text{pinion}}{\text{gear}} = \frac{17}{68} = \frac{1}{4}$

OBJECTIVES	NOTES TO TEACHERS

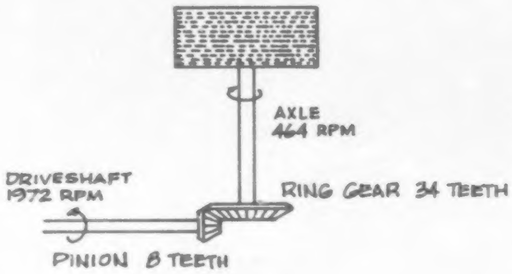
Example 3: Rear-Axle Ratios

The rear-axle ratio is the ratio of the number of teeth on the ring gear to the number of teeth on the pinion gear, or the ratio of the drive shaft speed to the rear axle speed.

$$\text{rear-axle ratio} = \frac{\text{ring gear teeth}}{\text{pinion gear teeth}}$$

$$= \frac{\text{drive shaft speed}}{\text{rear axle speed}}$$

Example: Rear-Axle Ratio



Question: Determine from the diagram the rear-axle ratio using 1) teeth ratio, 2) speed ratio.

Solution: 1) Rear-axle ratio

$$= \frac{\text{ring gear teeth}}{\text{pinion gear teeth}}$$

$$= \frac{34}{8}$$

$$= \frac{4.25}{1}$$

2) Rear-axle ratio

$$= \frac{\text{drive shaft speed}}{\text{rear axle speed}}$$

$$= \frac{1972 \text{ rpm}}{464 \text{ rpm}}$$

$$= \frac{4.25}{1}$$

OBJECTIVES

C. PROPORTION

The student should be able:

1. to explain the meaning of "proportion";
2. to solve a proportion for an unknown quantity when the other three are known.

NOTES TO TEACHERS

A proportion is a mathematical statement which equates two ratios.

Possible methods for the solution of a proportion are listed below. Students need not use more than one method.

Methods:

1) step-by-step using inverse algebraic operations

Example 1

$$\frac{x}{5} = \frac{3}{15}$$

$$\frac{x \cdot 5}{5} = \frac{3 \cdot 5}{15} \qquad \text{(mult. by 5)}$$

$$x = \frac{15}{15}$$

$$\underline{x = 1}$$

Example 2

$$\frac{5}{7} = \frac{15}{x}$$

$$\frac{5 \cdot x}{7} = \frac{15 \cdot x}{x} \qquad \text{(mult. by x)}$$

$$\frac{5x}{7} = 15$$

$$\frac{5x \cdot 7}{7} = 15 \cdot 7$$

$$5x = 105$$

$$\frac{5x}{5} = \frac{105}{5} \qquad \text{(divide by 5)}$$

$$\underline{x = 21}$$

OBJECTIVES | **NOTES TO TEACHERS**

2) clearing denominators, then using inverse operations

Example 1

$$\frac{x}{5} = \frac{3}{15}$$

$$\frac{x \cdot 15}{5} = \frac{3 \cdot 15}{15} \qquad \text{(mult. by 15)}$$

$$3x = 3$$

$$\frac{3x}{3} = \frac{3}{3} \qquad \text{(divide by 3)}$$

$$\underline{x = 1}$$

Example 2

$$\frac{5}{7} = \frac{15}{x}$$

$$\frac{5 \cdot 7x}{7} = \frac{15 \cdot 7x}{x} \qquad \text{(mult. by 7x)}$$

$$5x = 105$$

$$\frac{5x}{5} = \frac{105}{5} \qquad \text{(divide by 5)}$$

$$\underline{x = 21}$$

3) cross-multiplying

This method works for proportions, but students find it confusing when solving fractional equations involving more than one term on either side of the equation. Students using this method must realize the limitations.

D. DIRECT PROPORTIONS

The student should be able:

1. to explain what is meant by the term "directly proportional";

Two positive quantities which are in a proportion are said to be directly proportional if they are so related that an increase or decrease in one results in a corresponding increase or decrease in the other.

2. to set up a direct proportion between two variables and, given sufficient information, to solve for an unknown quantity;

Students should be encouraged to apply the five-step procedure summarized as follows:

1) Read the problem to determine the two quantities to be compared and represent them with symbols.

2) Create a ratio for one variable and then for the other variable in the same order.

3) Equate the two ratios to form a proportion.

4) Substitute the known quantities.

5) Solve the equation for the unknown quantity.

Example 1:

The mass of a steel bar is directly proportional to its length. If a bar 1 m long has a mass of 10 kg, how long is a similar bar having a mass of 18 kg?

Solution:

	First Bar	Second Bar
Step 1:	Length = ℓ_1	Length = ℓ_2
	Mass = m_1	Mass = m_2

Step 2: Ratio of lengths = $\frac{\ell_1}{\ell_2}$

Ratio of masses = $\frac{m_1}{m_2}$

Step 3: $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2}$

Step 4: $\frac{1}{\ell_2} = \frac{10}{18}$

Step 5: Solve for ℓ_2

$\ell_2 = 1.8$

Hence, the second bar is of length 1.8 metres.

Example 2:

If a service station gas pump can deliver 36 L of gasoline in one minute, how long will it take to fill a car's gas tank if it requires 84 L?

Step 1: Let A_1 = amount delivered in 1 minute

A_2 = amount to fill gas tank

T_1 = time to deliver A_1

T_2 = time to deliver A_2

Step 2: Ratio of amounts = $\frac{A_2}{A_1}$

Ratio of times = $\frac{T_2}{T_1}$

Step 3: $\frac{A_2}{A_1} = \frac{T_2}{T_1}$

Step 4: $\frac{84}{36} = \frac{T_2}{1}$

Step 5: $\frac{84}{36} \times 1 = T_2 \times 1$

$2.3\overline{3} = T_2$

Therefore, time to deliver 84 L is $2\frac{1}{3}$ or $2.3\overline{3}$ or approximately 2.33 minutes.

OBJECTIVES

NOTES TO TEACHERS

3. to determine the size of a real object given the scale and the scale drawing, and to find the size for a scale drawing given the scale and the size of the real object;

Note: In a scale drawing, the ratio is always drawing size to object size.

Example:

An object is represented by a scale drawing 3.8 cm by 1.5 cm with a scale of 1:3. What is the actual size of the object?

Solution:

Equate the scale ratio to a ratio of drawing size and object size for both height and width.

Thus: $\frac{1}{3} = \frac{3.8}{x}$ $\qquad$ $\frac{1}{3} = \frac{1.5}{y}$

$x = 11.4$ $\qquad$ $y = 4.5$

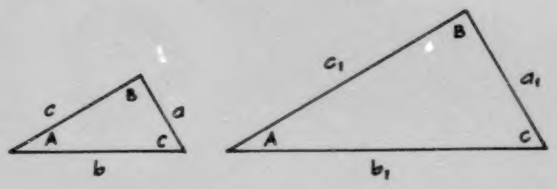
The actual object is 11.4 cm by 4.5 cm.

Students could be encouraged to measure familiar objects and to construct simple scale drawings.

4. to apply proportions to problems involving similar triangles.

For any given question, the two similar triangles should have the same orientation.

Example 1



For similar triangles, the ratios of corresponding sides form a direct proportion when equated.

OBJECTIVES	NOTES TO TEACHERS
	Thus: $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$ OR $\frac{a_1}{a} = \frac{b_1}{b} = \frac{c_1}{c}$

If $a = 3$, $c = 4$, and $b = 5$, find a_1 and c_1 if $b_1 = 7$.

$$\frac{a_1}{a} = \frac{b_1}{b}$$

$$\frac{a_1}{3} = \frac{7}{5}$$

$$\underline{\underline{a_1 = 4.2}}$$

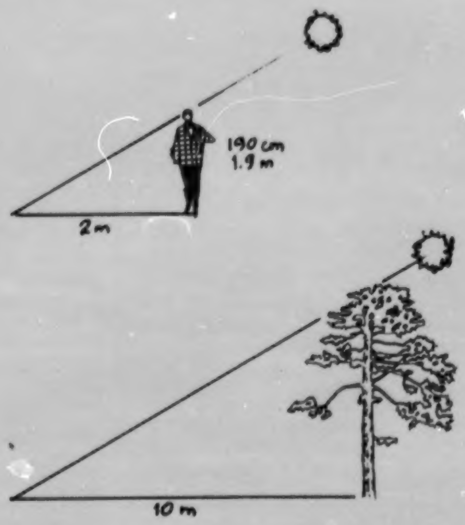
$$\frac{c_1}{c} = \frac{b_1}{b}$$

$$\frac{c_1}{4} = \frac{7}{5}$$

$$\underline{c_1 = 5.6}$$

Example 2:

A person 190 cm tall casts a 2.0 m shadow. If a nearby tree casts a shadow 10 m long, how high is the tree?



$$\frac{h_2}{h_1} = \frac{\ell_2}{\ell_1}$$

$$\frac{h_2}{1.9} = \frac{10}{2.0}$$

$$h_2 = \frac{10 \times 1.9}{2.0}$$

$$h_2 = 9.5$$

Therefore, the height of the tree is 9.5 m.

In addition, the principles of similarity could be extended to rectangles and squares.

OBJECTIVES

E. INVERSE PROPORTIONS

The student should be able:

1. to explain what is meant by the term "inversely proportional";
2. to set up and to solve inverse proportions.

NOTES TO TEACHERS

Two positive quantities which are in a proportion are said to be inversely proportional when they depend on each other in such a way that one increases when the other decreases.

Students should be encouraged to apply the five-step procedure summarized as follows:

1. Read the problem to determine the quantities to be compared and represent them with symbols.
2. Write a ratio for one variable in one order and for the other variable in the opposite order.
3. Equate the two ratios to form a proportion.
4. Substitute the known quantities.
5. Solve the equation algebraically for the unknown quantity.

Example 1:

The rate of vibration of a guitar string under constant tension varies inversely with the length of the string. The length can be varied by pressing the guitar string against the frets at various positions along the guitar neck.

If a guitar string 50 cm long vibrates 440 times per second, what is the length which will make the string vibrate 880 Hz?

Note: 1 hertz = 1 vibration cycle per second. The abbreviation for hertz is Hz.

Solution:

Step 1: Let F_1 = vibrations at ℓ_1 (440 Hz)

F_2 = vibrations at ℓ_2 (880 Hz)

ℓ_1 = length at 440 Hz

ℓ_2 = length at 880 Hz

Step 2: vibration ratio = $\frac{F_1}{F_2}$

length ratio = $\frac{\ell_2}{\ell_1}$

Step 3: $\frac{F_1}{F_2} = \frac{\ell_2}{\ell_1}$

Step 4: $\frac{440}{880} = \frac{\ell_2}{50}$

Step 5: $440 \times 50 = 880 \times \ell_2$

$$\frac{440 \times 50}{880} = \ell_2$$

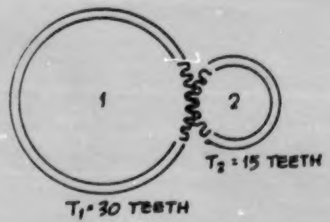
$$25 = \ell_2$$

Length of string is 25 cm.

Example 2:

The speed ratio of gears is inversely proportional to their teeth ratio.

Two gears with 30 and 15 teeth respectively are in mesh. If the large gear has a rotary speed of 100 rpm, find the rotary speed of the smaller gear.



Solution:

Let S_1 = speed of large gear

S_2 = speed of small gear

Speed ratio = $\frac{S_1}{S_2}$

Teeth ratio = $\frac{T_1}{T_2}$

$$\frac{S_1}{S_2} = \frac{T_2}{T_1}$$

$$\frac{100}{S_2} = \frac{15}{30}$$

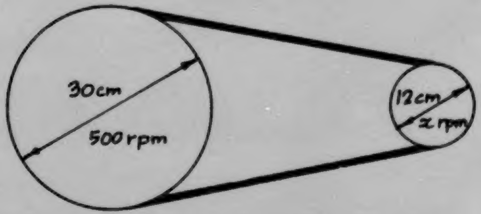
$$100 \times 30 = 15 \times S_2$$

$$200 = S_2$$

Example 3:

When two pulleys are belted together, their rotation in revolutions per minute varies inversely to the pulley size.

Find the rpm of the small pulley shown in the accompanying diagram.



OBJECTIVES

NOTES TO TEACHERS

Solution:

$$\frac{\text{small pulley diameter}}{\text{large pulley diameter}} = \frac{\text{large pulley rpm}}{\text{small pulley rpm}}$$

$$\frac{12}{30} = \frac{500}{x}$$

$$x = \frac{30(500)}{12}$$

$$x = 1250$$

The small pulley turns at the rate of 1250 rpm.

Principles of inverse proportion may be applied to a wide variety of problems involving the operation of pulleys in any of the following: automobiles (relation of engine crankshaft pulley to alternator pulley or power steering pulley), air conditioners (pulley speeds and sizes), table saws, lathes, belt-driven water pumps, grain augers, and other farm and industrial equipment.

Students may discover that this proportion applies to pulley circumference as well as diameter.

It is suggested that complicated pulley trains be treated as enrichment at this time.

Op.VII. PERCENT CALCULATIONS (1 module)

The three basic types of percent problems are differentiated on the basis of the unknown -- whether it be the amount, the percent, or the base. Although it may not be necessary for students to identify the various types, it is useful for them to do so. Also, identification is a benefit to teachers in preparing work sheets or selecting questions for assignments.

Included in each type are one-, two-, or more-step problems. It is suggested that the majority of time be spent on one-step problems of each type. More difficult problems can be introduced as enrichment or when proficiency in all three types of one-step problems has been gained. Exercises should contain a mixture of the three types of problems.

Method of Solution

Of the various methods for solving percent problems -- equation, one percent, or proportion -- the first, or equation method, is used in this guide. It is suggested that students use the following equation for all percent problems:

$A = R \times B$ where A = Amount or Percentage
R = Percent Rate
B = Base

For example, $20 = 40\% \times 50$

Students must identify the three variables of the particular problem and then solve the equation for the unknown.

Definitions and Explanations

In $A = R \times B$, the mathematical symbols are defined as follows:

A = Amount. Some textbooks use P (for percentage). "A" has been chosen for use here because of the tendency of students to confuse the term "percentage" with the number containing the % symbol.

R = Rate or Percent Rate. This term contains the symbol "%" and usually must be changed to its decimal fraction or common fraction equivalent before computation can occur.

B = Base. This is the number or quantity on which the percent rate is based. The ratio of the amount to the base is equivalent to the percent rate.

In translating an English sentence into mathematical symbols, the word "is" is translated as the symbol "=" and the word "of" is translated as the symbol "x."

Types of Percent Problems

1. Amount is unknown.

   15% of 40 is ???

   $A = R \times B$

   $A = 15\% \times 40$ (change 15% to a decimal fraction or a common fraction)

   $A = 0.15 \times 40$

   $A = 6$

   The most common example of this type of percent problem occurs in finding the provincial sales tax on a retail purchase.

2. Percent Rate is unknown.

   15 is ??% of 40

   $A = R \times B$

   $15 = R \times 40$

   $\frac{15}{40} = \frac{R \times 40}{40}$ (change $\frac{15}{40}$ to a percent rate)

   $37.5\% = R$

   Prior to solving problems of this type, proficiency in changing common fractions to decimal fractions and percent rates is necessary. A chart can be useful in developing proficiency.

   Example:

Common Fraction	Decimal Fraction	Percent Rate
$\frac{15}{40}$		
▨	0.8	
▨		20%

A common example of this type of percent problem occurs when converting the score on a test to a percent rate.

3. Base is unknown.

40 is 15% of ???

$A = R \times B$

$40 = 15\% \times B$ or $40 = 0.15 \times B$

$\frac{40}{0.15} = B$

$266.\overline{6} = B$

This is the least common of the three types of percent problems and the one with which students usually have the greatest difficulty.

Two-Step Problems

Once students have become proficient in recognizing which number in a problem represents the "amount," the "percent rate," and the "base" in one-step problems, they can be exposed to more complicated two-step problems. Such questions represent a significant increase in the level of complication for students.

Examples: "A" unknown

1) Find the total cost of an article priced at $6.80 if the sales tax rate is 5%.

Step 1: Find sales tax amount. $A = 5\% \times \$6.80$

$= \$0.34$

Step 2: Add sales tax amount to regular price.

$$\begin{array}{r} \$6.80 \\ .34 \\ \hline \$7.14 \end{array}$$

The total cost is $7.14.

2) Find the new population of Bluesville if its population decreased by 25% from 8400.

Step 1: Find decreased amount. $A = 25\% \times 8400$

$= 2100$

Step 2: Find new population. $8400 - 2100 = 6300$

The new population of Bluesville is 6300.

Examples: "R" unknown

1) The number of wigglegibbets sold increased from 52 000 to 58 240. What is the percent rate of increase?

Step 1: Find amount of increase. 58 240 - 52 000 = 6240

Step 2: Find the % increase. 6240 = R x 52 000

12% = R

2) The number of students in Room 115 decreased from 32 to 24 over the semester. What is the percent rate of decreas

Step 1: Find amount of decrease. 32 - 24 = 8

Step 2: Find the % decrease. 8 = R x 32

25% = R

Examples: "B" unknown

1) The new school enrollment in Oak Park School is 840 which represents a 40% increase over last year's enrollment. What was last year's enrollment?

Step 1: Recognize that 840 is 100% + 40% = 140% of last year's enrollment.

Step 2: Find 100% of last year's enrollment. 840 = 140% x B

600 = B

Last year's enrollment was 600.

2) The sale price of a watch at a "10% off" sale is $25.20. Find the regular price.

Step 1: Recognize that $25.20 is 100% - 10% = 90% of the regular price.

Step 2: Find the regular price. $25.20 = 90% x B

$28.00 = B

The regular price of the watch is $28.00.

OBJECTIVES

The student should be able:

1. to differentiate the three types of percent problems;

2. to determine the amount given the base and the percent rate;

3. to determine the percent rate given the base and the amount;

4. to determine the base given an amount and its percent rate;

5. to solve percent problems which occur in various areas of business, trade, or industry.

NOTES TO TEACHERS

In this objective, only one-step problems should be presented.

Students could cite examples of each type of percent problem, or they could be given a list of questions in which they would have to identify the unknown.

Example:

In each of the following, identify the unknown as amount, percent rate, or base:

1. 15% of 48 is ?

2. 15 is ? % of 48.

3. 48 is 15% of ?

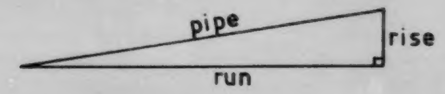
"A" unknown (one-step): If 8% of 3150 pipe elbows were faulty, how many had to be discarded?

"R" unknown (one-step): If 523 sheets of plywood were ruined in a flood, determine the percent rate of the loss out of a total of 3780 sheets.

"B" unknown (one-step): Out of a total number of hogs, a 32% loss meant that 64 hogs were lost. Find the total number.

Examples:

1) If the rise of a pipe is 1.5% of a run, determine the amount of rise for a run of 0.625 m. (Express the answer in m and in cm.) ["A" unknown, one-step]



2) Express, as a percent rate, the gradient of a roadway when the road rises 3.5 m in a run of 1.25 km. ["R" unknown, one-step]

3) If a mechanical device delivers 14.5 kJ of energy when 19.8 kJ are applied, what is the efficiency rating of the device? ["R" unknown, one-step]

4) Calculate the percent rate of the error when a speedometer reads 53 km/h if the correct reading is 48 km/h. ["R" unknown, two-step]

5) How many bricks must be purchased if 2850 are required for a job and 7% of those bought will be faulty (waste)? ["B" unknown, two-step]

6) An article priced at $4.80 has a 5% sales tax added to it. Find its total cost. ["A" unknown, two-step]

Op.VII. GRAPHING (1 module)

One of the most effective methods of communicating data is a well-constructed graph. Government agencies, corporations, organizations, and individuals publish pictorial representations of data to give the public a rapid and forceful comparison between two or more quantities or to lend support to their arguments.

Because graphs may contain a wealth of information with just a few bars or lines, and because graphs can be designed to prejudice the reader, it is important to read and to evaluate critically the information portrayed by a graph.

One of the best ways to learn to understand graphs is to construct various types.

In general:

- the graph should be self-contained and self-explanatory;
- the title should be clear and concise;
- the data on the axes should be identified;
- the lettering and digits should be legible; and
- there should be a break in the axis if the scale does not begin at zero.

Graphs related to the experiences of students will be more meaningful than those that are not. To most grade 10 students, graphing daily gas meter readings will be more relevant than graphing the Gross National Product of Canada over a five-year period.

Students may clip graphs from newspapers and magazines for a class collection.

Time and Newsweek magazines, atlases, economics and geography books, Canada Year Book, and Statistics Canada publications are excellent sources of graphs and data suitable for graphing. Other possible sources include: Agriculture Extension Offices and the Teachers' Investment and Housing Cooperative publications.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to construct and to interpret a bar graph;	Bar graphs are frequently used to compare the size of quantities: Examples could include: - daily egg production by cage or barn

- outside temperatures at a specific time of day (A thermometer could be mounted outside the classroom window and read during class.)

- the individual game scores of a basketball player during the season

- the number of T.V.'s in the homes of students — black and white as well as color

- the heights of ten Canadian mountains

- the lengths of the ten longest Canadian rivers

- the distribution of grade 10 marks in one subject or in all subjects during a term or semester

- student absences on each day of the week

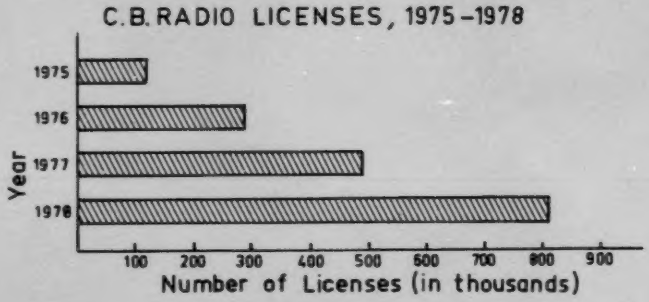
- the enrollment in a specific grade over a period of years

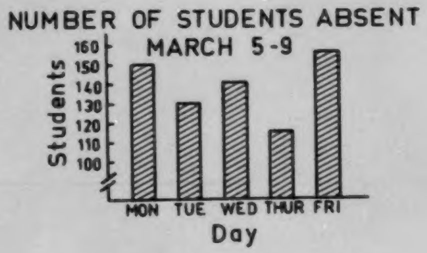
- salaries paid in various occupations (This information may be available from your local salary negotiations chairpersons.)

- the number of different colored Smarties in a box

- the price of gold or some other commodity over a period of time

Bar graphs may be horizontal or vertical.


C.B. RADIO LICENSES, 1975-1978


NUMBER OF STUDENTS ABSENT MARCH 5-9

OBJECTIVES

2. to interpret various kinds of bar graphs;

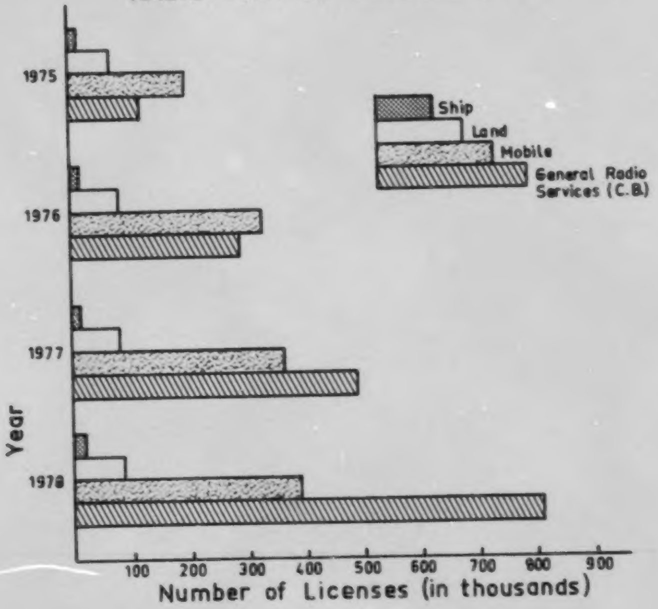
NOTES TO TEACHERS

Types of bar graphs include:

1) Multiple Bar Graph

A multiple bar graph illustrates two or more sets of data for each time category.

Example:



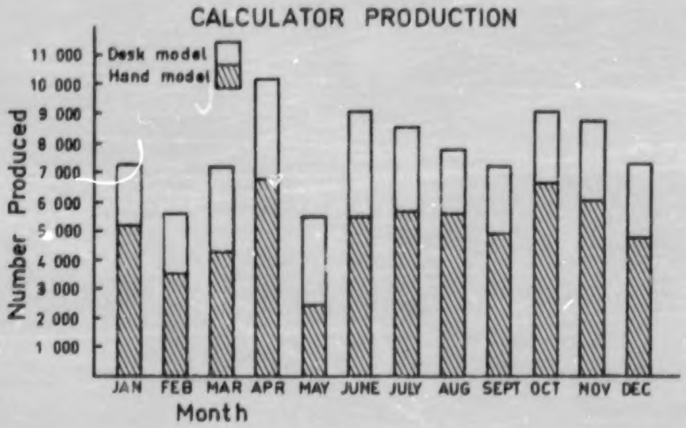
RADIO STATION LICENSES, 1975-1978

OBJECTIVES	NOTES TO TEACHERS

2) Component Bar Graph

A component bar graph stacks the various sets of data so that the top of the bar is the total of the sets.

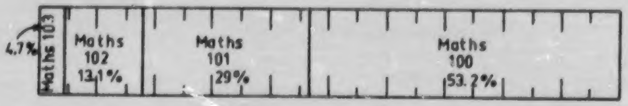
Example:



3) 100% Bar Graph

The 100% bar graph is a rectangular graph with each set of data occupying a percent of the total area of the rectangle.

20 381 Manitoba Grade 10 Students Enrolled in Mathematics
1976-77



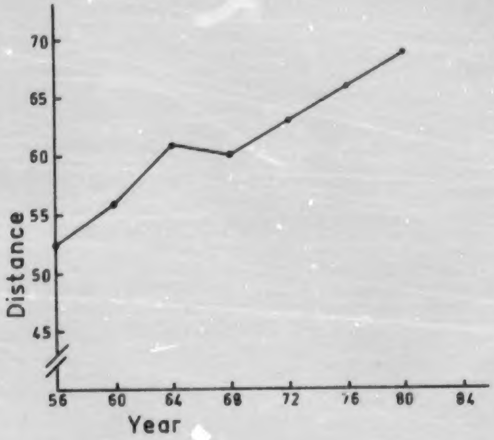
OBJECTIVES

3. to construct and to interpret a single line graph;

NOTES TO TEACHERS

The line graph is used to indicate changes between two sets of data, one of which is usually time. Frequently, the line graph is used to predict future results.

WOMEN'S OLYMPIC JAVELIN THROW



Examples could include:

- Olympic winning distances in a throwing event such as javelin, over a period of years
- average typing speed of grade 10 students over a regular period of time
- consumption of electricity in a school, taken from hydro billings over a two-year period
- expenditures on electricity at a student's home as taken from utility bills
- gas meter readings at a school or home as taken from utility bills
- daily water meter readings in a home over a one-week period
- enrollment by grade in the school
- daily growth of a potted plant such as a bean in the classoom
- readings of a photometer, at hourly intervals, for two flashlights using different brands of batteries

OBJECTIVES

4. to interpret various types of line graphs;

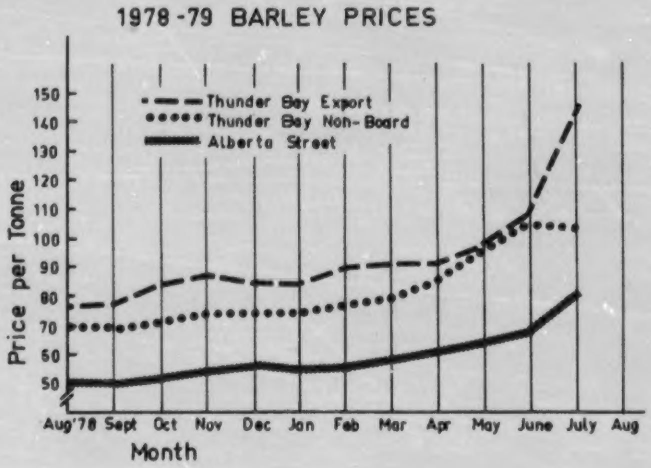
NOTES TO TEACHERS

Types of line graphs include:

i) Multiple Line Graph

The multiple line graph shows two or more sets of data over the same period of time.

Example:


1978-79 BARLEY PRICES

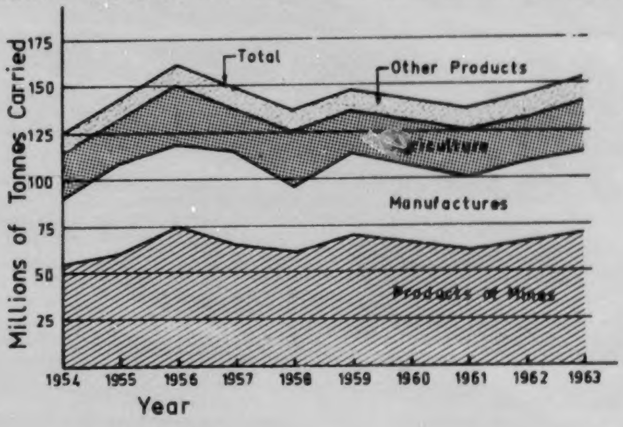
OBJECTIVES | **NOTES TO TEACHERS**

2) Component Line Graph

In a component line graph, each component part makes up part of the whole. The top line represents the total of all the components.

Example:

FREIGHT TONNAGES CARRIED BY CANADIAN RAILWAYS



Courtesy of Elements of Mathematics for Secondary Schools, Dean and Moore, Holt, Rinehart & Winston, 1966.

5. to construct and to interpret a circle graph;

The circle graph, or pie chart, is used to show the relation of parts to the whole and to each other.

Examples could include:

- expenditures of the student council on various council activities such as dances, yearbook, sports, canteen, hayrides, etc.
- the town budget allotments for protection, transportation, streets, education, and other services (This information is available from municipal and town offices.)
- an analysis of a school's operating budget for expenditures on paper, library, etc.
- expenditures by a school team on uniforms, referees, advertising, etc.

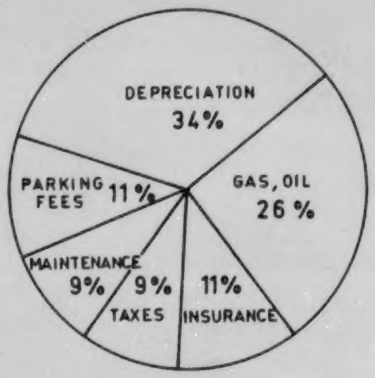
OBJECTIVES

6. to construct and to interpret a pictograph;

NOTES TO TEACHERS

Example:

COSTS OF OPERATING A CAR



Courtesy of Mathematics for Daily Use, M.P. Hayden, et al., Doubleday Canada Ltd., 1980.

The data on a pictograph is represented symbolically. The pictograph sometimes sacrifices accuracy for an overall impression and may be designed as an advertisement.

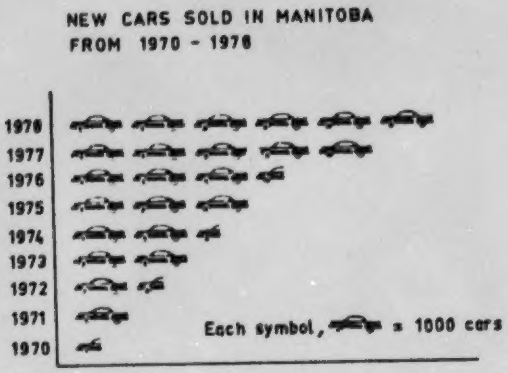
Example could include:

- municipal taxes levied on a household over a period of years (This information may be available from property tax notices.)
- the annual output of wheat, pulpwood, or some other product
- the population of the provincial capital cities

OBJECTIVES

NOTES TO TEACHERS

Example:



7. to recognize distortion in graphs.

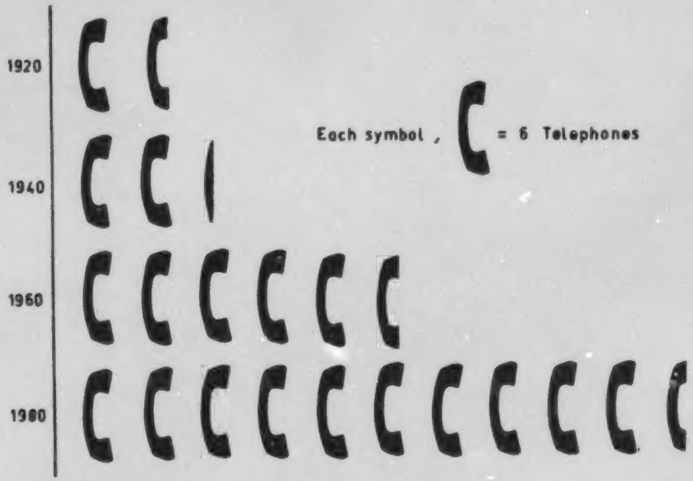
Graphs may be drawn so as to distort information. For example, removing the bottom of a graph, stretching the vertical axis, or drawing the symbols of a pictograph disproportionately can change the visual impression dramatically. Even the wording and location of the title of a graph can affect the impression.

427

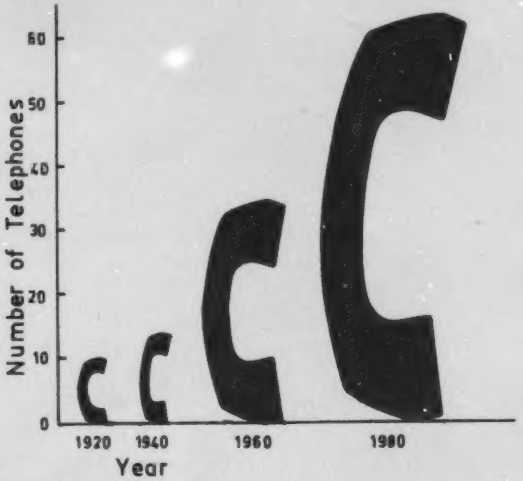
428

Example I: Telephones in Canada.

NUMBER OF TELEPHONES PER 100 POPULATION IN CANADA



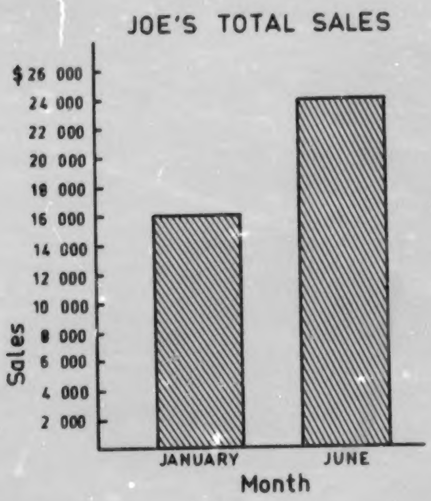
NUMBER OF TELEPHONES PER 100 POPULATION IN CANADA



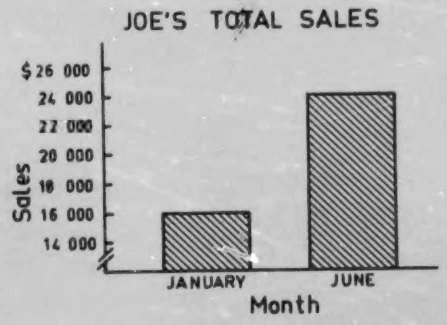
Example 2: One Man's Sales

Salesman Joe graphed his total sales for January and June.

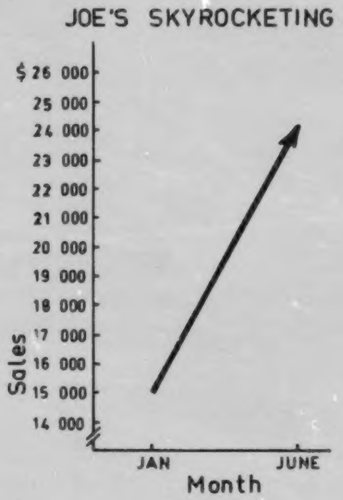
Graph A: Salesman Joe graphed his total sales for January and June the way he had been taught in Math 101.



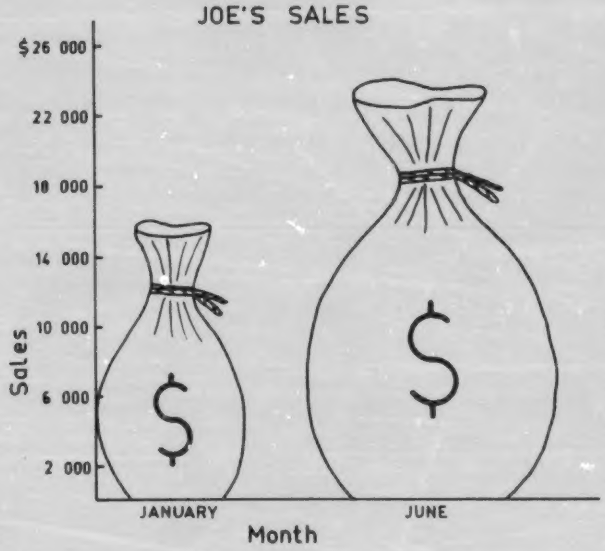
Graph B: In order to impress his boss, Joe removed the bottom of the graph. The scale did not begin at zero but it certainly showed his increased sales volume in a better light.



Graph C: Still, the effect was not pronounced enough, so Joe stretched the vertical axis and left off the bottom of the graph. A slight title change didn't hurt either.



Graph D: Joe's final effort was a pictograph. Since the ratio of June sales to January sales was 3:2 he had to draw the second money bag 1½ times as tall as the first. To keep the proportions accurate he also increased the width of the second bag by 1½ times. The numbers still say 3:2, but the area says 9:4. However, since a money bag is three-dimensional, it actually leaves the impression that his sales have increased $(\frac{3}{2})^3$ or approximately 340%, when in fact his sales had increased only $8000 over his $16 000 in January for an actual increase of only 50%.



Resource: The Time charts by Nigel Holmes, printed in Time magazine, are well-known examples of eye-appealing, impressive, and well-prepared graphics. Many of Mr. Holmes' works are multiple line graphs, bar graphs, and pictographs all in one.

Op. IX. ESTIMATION (½ module)

The ability to determine whether a solution to a mathematical problem is reasonable or realistic is a vital mathematical skill in a society advocating the use of calculators. Students have a tendency to believe that the number that appears on a calculator after an operation has been performed is correct. It is important for students to realize that such a belief is unwarranted since incorrect input/operations will produce wrong and unreasonable results. In order to develop student awareness of reasonable answers, teachers must focus on estimation. Increased attention should be placed on estimations of time, of space, of cost, and of number.

Since estimation is difficult and is possible only after concepts are understood, the purpose of this unit is to teach the concepts associated with estimation. Teachers should use uncomplicated examples to establish the ideas which then should be integrated into the remainder of this and other OI math courses. The objective is to give students experience in estimation and not to obtain a correct/accurate final answer.

The suggested method of handling this unit is as follows:

1. Spend an initial period of time, say 3 or 4 class periods, giving students experience in estimation. Concentrate on estimation involving time and space rather than arithmetic operations.

2. Integrate all remaining time throughout the remainder of the course.

One problem that will arise in this section is judging the order of magnitutde for a reasonable estimate. This will constantly pose a problem and it is suggested that each teacher establish limits for acceptable answers. No such rules have been established in this guide.

Discussion of the parameters necessary for making reasonable time estimates should be encouraged. A major aim of the type of exercise found below is to encourage students to read or listen and then to follow directions.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to estimate the time required for a given task;	Students should be required to estimate, to measure, and to compare the two results in situations such as the following: 1) walking from Point A to Point B. (Individual students could be asked to estimate the time to go from A to B and to return. Then, they could be timed by other members of the class when they actually walk the predetermined route.)

OBJECTIVES | **NOTES TO TEACHERS**

2) completing a specified number of arithmetic exercises using a calculator or paper and pencil techniques. (Students should estimate the time it will take them to finish the exercises, then do the calculations as accurately and as quickly as possible. Teachers should stress the need for accuracy in such activities as speed tests, mental arithmetic, etc.)

3) using public transportation

4) perceiving the passage of time under different conditions (Students could be asked to estimate time both while actively participating in an activity or while acting as a spectator. Comparison of the two estimates should be made. Students should realize that time seems to pass more quickly when the mind is occupied.)

Students should learn to maximize productivity in the classroom and thereby make time outside class available for leisure, social or job-related activities. Wise use of time, in particular, short periods of time, should be encouraged. Students should learn what they can expect to accomplish in a specified period of time.

Practical examples, such as the following, could be employed:

1) check-out time at a supermarket
2) time to shingle a roof
3) time for professional carpet layers to carpet a room
4) time to list, using the telephone directory, all the grocery stores in Winnipeg
5) time to travel from Brandon to Winnipeg including a consideration of factors that affect travel time.

2. to make reasonable spatial estimations;

Students should estimate, record the estimate, measure, record the measure, and then compare the estimate with the measure in physical measurement situations.

Students could concentrate on a number of the following:

1) metric measures of length (Pacing, handspans, etc., could be used to assist students in making reasonable estimates for distance. Such estimates should begin with the smaller units such as mm and cm and then should proceed to larger units such as m and km.)

2) measure of perimeter, area, and volume (Different shaped figures should be used. For example, have students compare the volume of a tall rectangular solid with that of a more compact one.)

OBJECTIVES

NOTES TO TEACHERS

3) height and mass (weight) calculations

4) shoe sizes

5) the number of sheets of paper in a given stack, or the number of pennies required to make a stack 4 cm high

6) the number of students capable of standing on a square metre

7) the number of objects, people, or things in a group

8) capacity

9) office or warehouse layout including aisle size

10) shop layout for beauty culture - chair width, sink size, etc.

11) height of a stack of one million dollar bills; the length of time it would take to spend it

It is not enough to say, "estimate." Systematic and regular effort must be given to developing the skill of spatial estimation. The following steps are suggested:

Step 1: Have students select an appropriate standard in terms of which the following comparisons can be made: bigger, longer, shorter, heavier, thicker, taller, etc.

Step 2: Have students practice estimation and direct measurement of familiar objects.

Step 3: Shift the emphasis, and have students estimate objects which are not in view. Follow this with actual measurement of the objects.

Step 4: Finally, reverse the process and encourage students to give examples of objects which are approximately of a given length, area, volume, size, etc.

Games could be used to reinforce the process of estimation.

3. to estimate arithmetic calculations;

Note: Students traditionally have had difficulty with the estimation of arithmetic computations. It appears that paper and pencil algorithms discourage estimation in arithmetic because of the desire for exact answers. Teachers are encouraged to initiate practice sessions designed solely for estimating answers. Rather than have students check estimates with paper and pencil calculations, discuss with them how estimates can be made. Encourage them to use mental arithmetic in conjunction with their calculators.

OBJECTIVES

4. to estimate the total cost including sales tax and change associated with consumer buying.

NOTES TO TEACHERS

If students are to estimate arithmetic computations, effort must centre on:

1) betweenness and the analysis of number relations (Teachers are advised to keep this analysis at a very general level. For example, students must realize that 2.4 is between 2 and 3 and slightly closer to 2.)

2) rounding numbers (Students should be able to round numbers to the nearest 100, 10, 1, 0.1, and 0.01.)

   Example: Round 234.9 to the nearest hundred.

   Answer: 200

3) multiplying/dividing by powers and multiples of 10

4) multiplying/dividing by values between 0 and 1 (Students tend to believe that multiplying increases the value whereas dividing decreases the initial value. They must realize that when multiplying or dividing by numbers between 0 and 1 the opposite is true.)

5) mental calculations (To aid students in making mental estimate calculations, arithmetic exercises, rapid calculation drills, number games, and the tachistoscope could be used.)

Teachers may wish to have student work on estimations involving money prior to doing work on objective #3.

The complexity of calculations will be determined by student ability.

Use consumer applications. Examples could include:

1) the total cost of hockey equipment
2) the number of cubic metres of air for each occupant of a room
3) the cost of gasoline to drive "x" kilometres
4) other costs of running an automobile
5) the cost of supplying basketballs for a school

Examples:

1. What is the cost of 1 can of soup if priced at 3/76¢?
2. What is the cost of 7.2 kg of produce "M" at 2 kg for $0.97?

Op.X. MATHEMATICAL EXPERIMENTS (½ module)

A math lab may be viewed as a mode of classroom management in which students, working individually or in small groups, do experiments in mathematics. Each of these experiments presents the student with activities (usually with physical apparatus) which have mathematical implications.

Some advantages of the math lab approach are the possibility of enhancing students' motivation to learn mathematics, the presence of a natural learning sequence of "concrete-to-abstract," and the opportunity to discover real-world applications of mathematics.

The math lab does require some planning, and it may take students time to get used to working in a freer situation. Perhaps one day every week or two could become the math lab day. Having students collaborate on experiments permits discussion and mutual assistance.

One way of evaluating a student's work is to have each member of a lab team write up the finished experiment, describing what was done, how it was done, and what the findings were.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to perform a mathematical experiment given written instructions;	Several experiments have been included as examples. Teachers could use these sample experiments or any others which they have or can obtain. Example 1: Ruler Dropping Experiment This is an experiment related to exponents and radicals. It is an attempt to modify an abstract approach and to shift toward some use of sensory impression. The experiment integrates mathematics with physics and biology and can be used as a metric activity. Materials: Metric rulers Objective: To measure student reaction time by measuring the distance a ruler falls before it is caught. Procedure: Students work in pairs with one ruler per pair. One student (A) holds a ruler vertically between his/her thumb and forefinger at the 30 cm end.

The second student (B) places his/her thumb and forefinger horizontal to the ruler at the 0 mark. The fingers are kept about 2 cm apart with the ruler midway between them. When student A releases the ruler, student B is to catch it between his/her fingers. Each time the ruler is dropped, the distance it falls before student B catches it is recorded in a table similar to the one shown below.

Students should switch positions after a number of trials.

Data Table

NAME	TRIAL				
	1	2	3	4	MEAN
A					
B					
⋮					

Sample Calculations:

A person with average reaction time should catch the ruler in approximately 15 cm.

Use $d = 490t^2$, where d is the distance in cm and t is the time in seconds.

These calculations indicate a practical use for solving equations and allow students to practice computing roots using calculators.

If the mean for a student is 16 cm, calculations would include:

$d = 490t^2$

$16/490 = t^2$

$t = \sqrt{\frac{16}{490}}$

$t \doteq$ 0.181 seconds which is the student's maximum reaction time

447

448

Example 2: Perimeter Experiment

Material: 36 unit cubes, graph paper

Objective: To make rectangles out of 36 unit cubes in as many ways as possible.

Procedure: The perimeter is recorded in a table similar to the following:

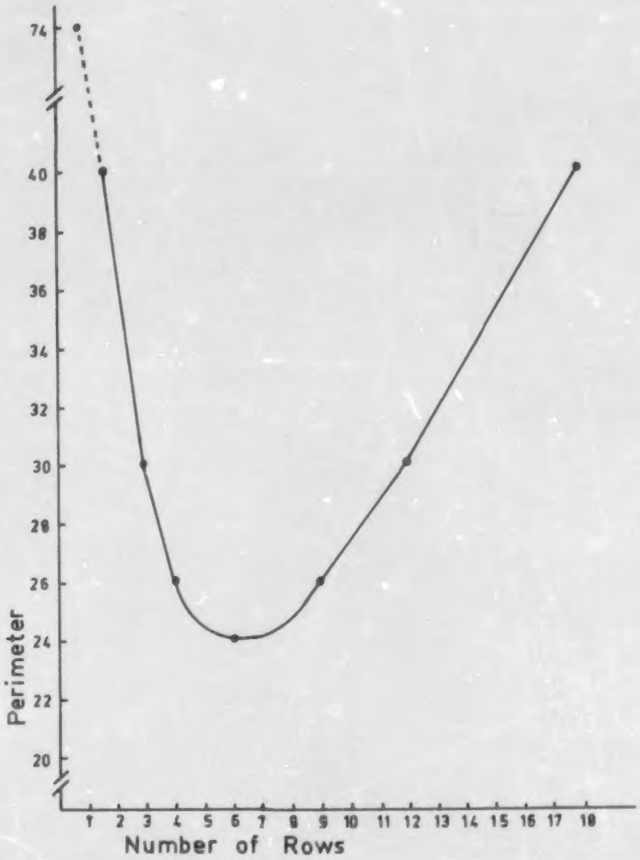
Number of Rows	Perimeter
1	74
2	40
3	30
4	26
5	
6	24
7	
8	
9	26
10	
11	
12	30
.	.
.	.
.	.
36	74

Knowledge of fractions and the area of a rectangle will permit students to fill in the gaps in the table.

OBJECTIVES

NOTES TO TEACHERS

Observations: Students can record their results on a graph similar to the following:



Note that the graph is hyperbolic, not parabolic.

Example 3: Finding the Volumes of Rectangular Solids, Cylinders, and Spheres

Materials: Metric ruler, overflow can, beaker, graduate cylinder, water, solid rectangle, solid cylinder, solid sphere, rectangular container, cylindrical container, round balloon, Plasticine.

Objective: To determine the volumes of a rectangular solid, a cylinder, and a sphere by first measuring their dimensions and then calculating the volume using a standard formula. The volumes so calculated are to be verified by a water displacement method.

Method 1: Measure the dimensions of the solid object with the ruler. Calculate the volume according to the given formula. Place the object in the water in the overflow can. Measure the volume of the water displaced with the graduate cylinder. How does the calculated volume compare with the volume obtained by the displacement of water?

Object	Dimensions	Volume Formula	Calculated Volume	Displacement Volume
rectangle		$V = l \times w \times h$		
cylinder		$V \doteq 3.14 \times r^2 \times h$		
sphere		$V \doteq \frac{4}{3} \times 3.14 \times r^3$		

Method 2: Measure the inner dimensions of the container with the ruler. Calculate the volume according to the given formula. Fill the container with water. Measure the volume of water in the container by using a graduate cylinder. How does it compare with the calculated volume?

Object	Dimensions	Volume Formula	Calculated Volume	Displacement Volume
rectangle		$V = l \times w \times h$		
cylinder		$V \doteq 3.14 \times r^2 \times h$		
sphere		$V \doteq \frac{4}{3} \times 3.14 \times r^3$		

OBJECTIVES	NOTES TO TEACHERS

Method 3: Make a rectangular solid from Plasticine. Find its volume. Roll it into a cylinder. Again calculate its volume. Repeat the procedure by rolling it into a sphere. How do all three volumes compare with each other?

Rectangular solid __________ Cylinder __________ Sphere __________

Example 4: The Tower of Hanoi

Materials: A piece of board about 15 cm by 30 cm with 3 pegs spaced about 15 cm apart, circular disks (usually between 5 and 10) with holes in their centres

Each of these disks must be a different size, arranged in descending order so that the largest disk is at the bottom and the smallest disk is at the top.

Objective: To transfer disks from one peg to another in a minimum number of moves according to the following rules:

- only one disk can be moved at a time;
- after a move, each smaller disk must be on top of a larger disk.

It is suggested that students begin with only 2 disks on one of the pegs. After they have successfully transferred these 2 disks, they may try the game with 3 disks, 4 disks, etc. Have them keep track of the number of moves required.

Number of Disks	Minimum Moves Required
1	__________
2	__________
3	__________
4	__________
5	__________
6	__________

Questions:

1) Estimate the number of moves necessary to move 10 disks.

2) Work out a formula which would give the number of moves required to transfer any number of disks.

3) If a person could move 1 disk per second, how long would it take to move a stack of 28 disks?

Solutions:

1) Answers will vary.

2) The minimum number of moves required to move "n" disks from one peg to another, using the given rules, is 2^n-1.

3) 1 min. = 60 sec.; 1 hr. = 60 min.; 1 day = 24 hrs.;
1 year = 365 days

so 1 year = 60 x 60 x 24 x 365

= 31 536 000 sec.

To transfer 28 disks at one transfer per second, it would take:

$2^{28} - 1 = 2.68 \times 10^8$ seconds

The time needed to transfer these disks would be:

$$\frac{2.68 \times 10^8}{31\ 536\ 000} \doteq \underline{8.5 \text{ years}}$$

In fact, to move 30 disks, it would take:

$$\frac{2^{30} - 1}{31\ 536\ 000} \doteq 34 \text{ years}$$

Additional experiment ideas include:

1) geometric exercises involving the construction of polyhedra

2) market research such as counting cars, products passing through a supermarket check-out, etc.

3) interest rate calculations, such as determining the length of time necessary to double a given principal at various interest rates.

Op.XI. SPATIAL RELATIONS (½ module)

The emphasis of this unit is to increase a student's ability to see and to manipulate geometric forms rather than to prove or to express written arguments. What is important is the visualization and perception of shape and movement in space.

OBJECTIVES

A. MOTION GEOMETRY

The student should be able:

1. to identify and to perform the basic transformations (reflection, rotation, translation) on both linear and curved geometric figures;

NOTES TO TEACHERS

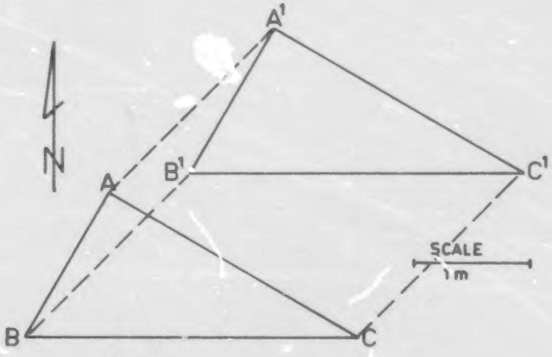
Basic to motion geometry is the movement or transformation of one set of points to another location on the same plane. In particular, "rigid transformation" is studied. In a rigid transformation, the size and shape of a figure (set of points) is unaltered; that is, no distortion occurs as a result of transformation.

All transformations are made up of a combination of three basic types of transformation: translation, rotation and reflection.

1) Translation -- All points are moved the same distance in the same direction.

Example:

Translate ΔABC 2 m NE.

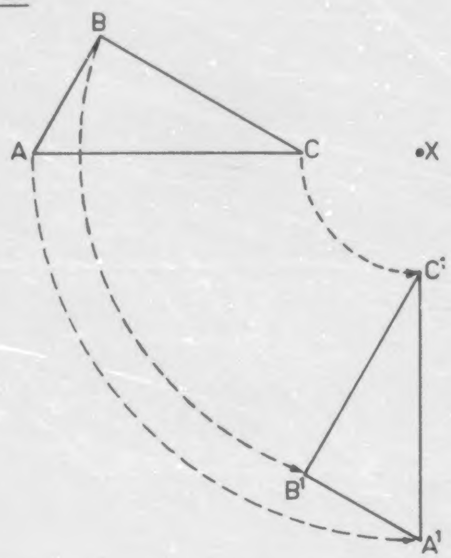


OBJECTIVES

NOTES TO TEACHERS

2) Rotation -- All points are turned by the same angle in the same direction around a fixed point or centre of rotation.

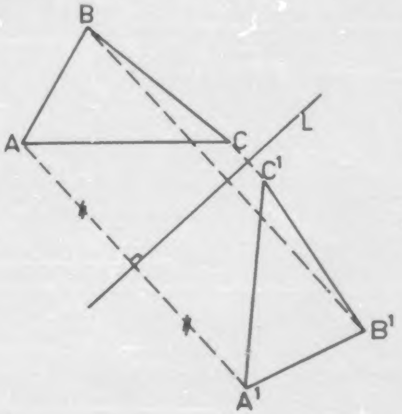
Example:



Rotate ΔABC 90° counter-clockwise around X.

3) Reflection -- All points are moved across a line of reflection to a location on the opposite side, the same distance from the line as their original location.

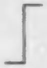
Example:

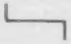


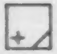
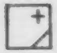
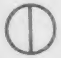
Reflect ΔABC across line L.

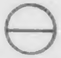
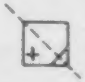
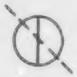
OBJECTIVES	NOTES TO TEACHERS

Types of problems could include:

1) is to as  is to what?

Solution:  (90° rotation)

2)  is to  as  is to what?

Solution:  (reflect  , and then reflect )

3) Describe a series of movements which will move figure 1 to the position a orientation of figure 2.

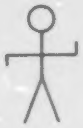


Figure 1

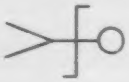


Figure 2

Solution: Reflect around vertical line in centre, then rotate 90° clockwise.

Note: Several answers may be possible to questions of this type.

OBJECTIVES

2. to identify lines of symmetry of figures and the reflections which will carry a figure onto itself;

3. to identify the centre(s) of rotational symmetry and the order of rotational symmetry.

NOTES TO TEACHERS

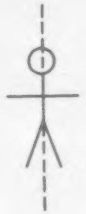
Note: There are two types of symmetry: symmetry about a line and symmetry about a point (rotational symmetry).

A line of symmetry divides a figure into two mirror images; that is, a reflection across the line of reflection puts the figure back on top of itself.

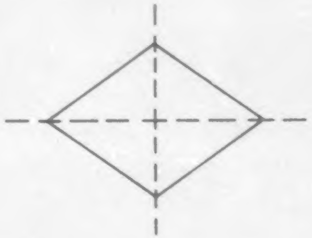
Example:

i) Identify the line(s) of symmetry in each of the following:

1.



2.

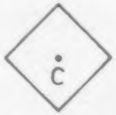


Examples could involve letters or numbers.

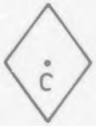
In rotational symmetry there is a centre of rotation with an angle and direction such that the figure lies on itself if rotated (around the centre) at that angle in a given direction.

The order of rotational symmetry is the number of times the rotation is repeated in order to give one full rotation to the figure.

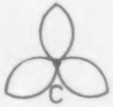
Examples:



Square - centre C, rotate 90°, 180°, 270°, or 360°
order = 4



Rhombus - centre C, rotate 180° or 360°
order = 2



Pinwheel - centre C, rotate 120°, 240°, or 360°
order = 3

OBJECTIVES

B. PATTERN AND PERSPECTIVE

The student should be able:

1. to lay out a pattern from which each of the following can be folded:

   a) cube,
   b) tetrahedron,
   c) rectangular solid;

2. to decide whether a given layout can be folded to give a cube, tetrahedron, or rectangular solid;

NOTES TO TEACHERS

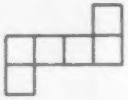
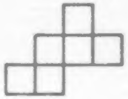
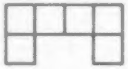
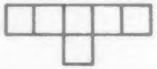
Layout of 3-D figures in 2-D:

The aim here is to increase the student's ability to visualize the relation between the 2-D figure and the 3-D object.

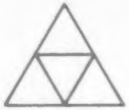
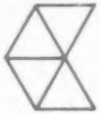
When laying out the faces of a 3-D figure for cutting in 2-D, the relation between the 2-D pattern which is to be cut and folded to give the final 3-D object is not unique.

Examples:

1) Which of the following can be folded to give a cube?

2) Which of the following can be folded to give a tetrahedron?

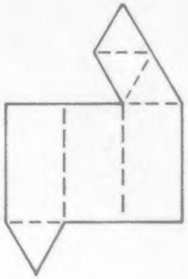



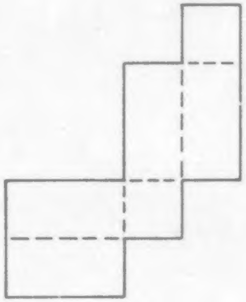
Are there any other forms which would give a tetrahedron?

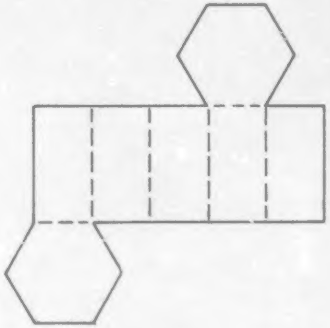
OBJECTIVES

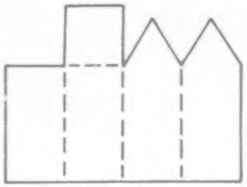
NOTES TO TEACHERS

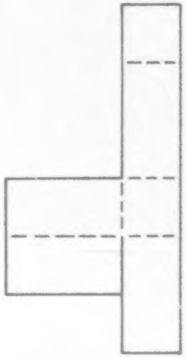
3) Which of the following can be cut on the solid lines and folded on the dotted lines to give a solid figure?

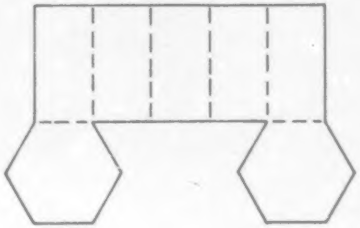












OBJECTIVES	NOTES TO TEACHERS

3. to make 1- or 2- point perspective drawings of
   a) cube,
   b) rectangular solid,
   c) prism,
   d) mixture of the above.

Note: In a two-dimensional representation, perspective creates the illusion of depth, or of a third dimension, by allowing the mind to interpret as solid a figure composed of plane lines. There are many types of perspective, but in this unit the focus is limited to 1- and 2-point perspective.

1-Point Perspective: The following figure uses 1- point perspective, since its face is essentially flat to the viewer.

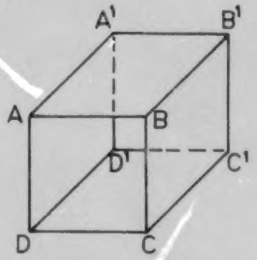


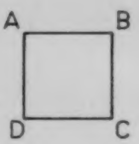
Fig. I

In figure I, it is not clear whether face ABCD or face $A^1B^1C^1D^1$ is closer to the viewer (see the section on illusion). If the viewer observes an actual cube, the lines AA^1, BB^1, CC^1, and DD^1 appear to get closer or converge, and to point to a position on the horizon. Further, in an actual cube A^1D^1, B^1C^1, A^1B^1, D^1C^1 appear to be shorter than AD, BC, AB, DC, because they are farther away. This is called diminution.

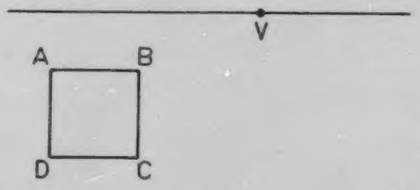
Step-by-step procedure for drawing a cube in 1-point perspective:

1. Draw the horizon line.

2. Draw the closest face of the object.



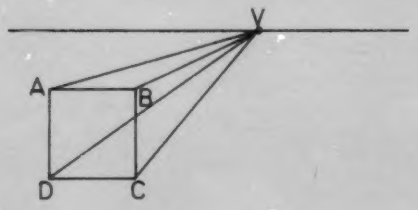
3. Mark a vanishing point, V, on the horizon. (It could be to the left or the right of the face drawn.)



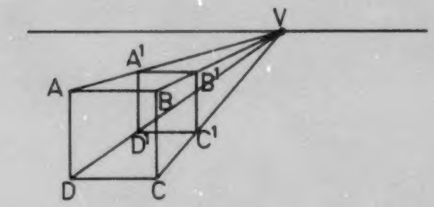
OBJECTIVES

NOTES TO TEACHERS

4. Draw lines from each corner of the face to the vanishing point. (If the object is assumed to be opaque, then the line DV should not be drawn like the others but could be represented on the diagram by a dotted line.)

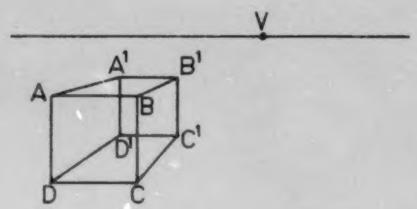


5. Draw edges of the back face parallel to corresponding edges of the front face. (A^1B^1, C^1D^1 are not drawn if the object is opaque, but could be represented by dotted lines.)



6. Erase continuation lines to the horizon.

7. Compare this figure with figure 1.



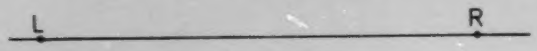
Note: This could be demonstrated in a series of overlays to show the development.

OBJECTIVES | NOTES TO TEACHERS

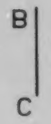
2-Point Perspective: Unlike 1-point perspective, in which horizontal lines converge on four sides only, 2-point perspective shows some convergence of the horizontal lines on all sides. Only the vertical edges do not appear to converge.

Step-by-step procedure for drawing a rectangular box in 2-point perspective:

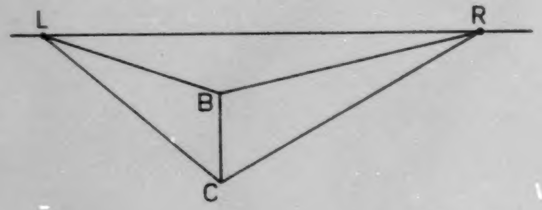
1. Draw the horizon line.



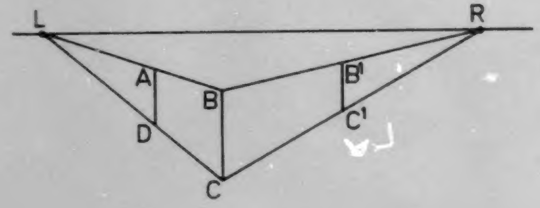
2. Draw the foremost vertical edge of the box, BC, and two vanishing points on the horizon -- on the left, L, and on the right, R.



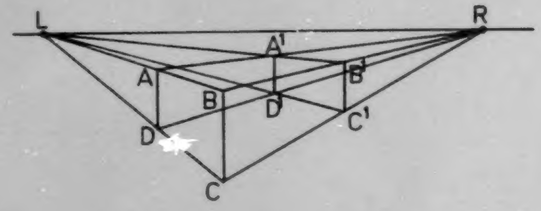
3. Draw lines from the top and bottom of the edge, BC to each vanishing point.



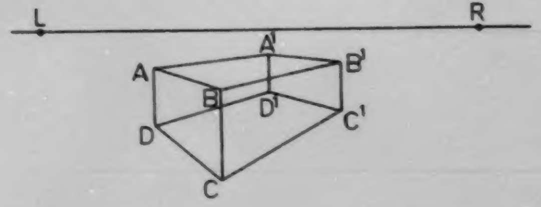
4. Draw the next two vertical edges of the box parallel to the front edge.



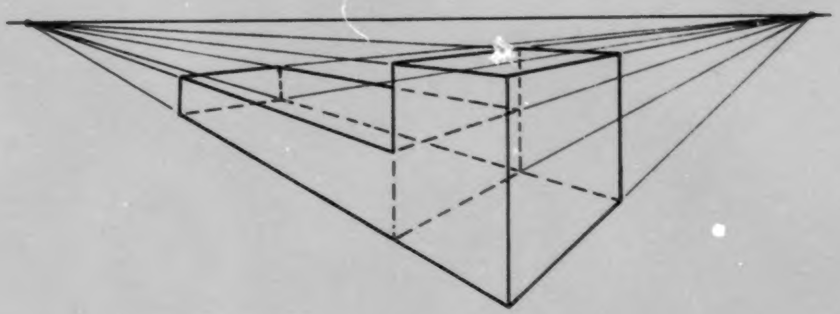
5. Draw lines from the corners D, A, C^1, B^1 to the vanishing points opposite them. Put in the vertical edge at the back, A^1D^1.
(C^{1}L, DR, and A^1D^1 would not be drawn if the object were opaque. Dotted lines could be used to represent these).



OBJECTIVES	NOTES TO TEACHERS
	6. Erase extra lines.



Note: This could be demonstrated in a series of overlays to show the development.



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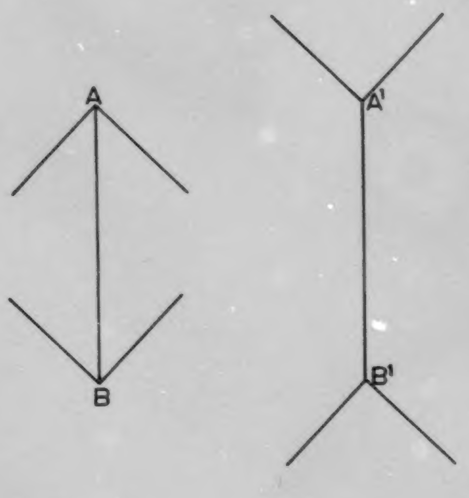
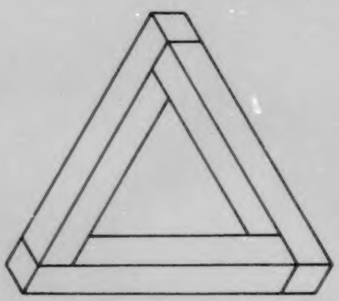
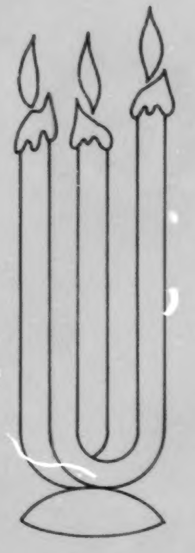


AB and A^1B^1 are the same length.





Enrichment Example:

3-Point Perspective: In objects of great height -- in a tall building, for example -- there appears to be vertical as well as horizontal convergence. To diagram this, a third vanishing point on the centre line is used for vertical lines.

C. ILLUSION

When seen, some figures can trouble the mind by creating contradictory mental images. In part, this occurs because a mental image is an interpretation of a figure, and not the figure itself. Figure I in Perspective, for example, has two equally viable interpretations as a 3-D object, one with ABCD as the closest face and the other with $A^1B^1C^1D^1$ closest.

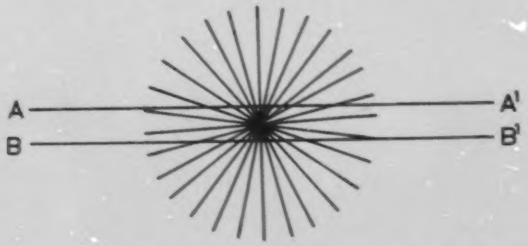
As the perspective examples in this section indicate, illusions can be useful and lend realism to drawings. But some illusions make drawings unrealistic. In this course, the object of examining illusion is to demonstrate that pictures can confuse the issue.

The student should be exposed to optical illusions.

The main objective is experience with and exposure to illusions, not proofs and formalism. Consider:

- the art work of M. C. Escher
- perspective in the illusion of a third dimension

Examples:



$AA^1 // BB^1$ - Both lines are straight.

OBJECTIVES

NOTES TO TEACHERS

Resources:

1. Escher, M.C. The Graphic Work of M. C. Escher. New York: Ballantine, 1971.

2. Evans, Larry. Three Dimensional Optical Illusions to Color and to Construct. San Francisco: Troubador Press, n.d.

3. Gardner, Martin. "Mathematical Games," Scientific American, Vol. 236, No. 1 (January 1977), pp. 110-121.

4. ______________. "Mathematical Games," Scientific American, Vol. 238, No. 1 (April 1978) pp. 16-33.

5. Gregory, Richard L. "Visual Illusions," Scientific American, Vol. 219, No. 5 (November 1968), pp. 66-76.

6. Kanizsa, Gaetano. "Subjective Contours," Scientific American, Vol. 234, No. 4 (April 1976), pp. 48-52.

7. Manitoba Department of Education. "Putting It In Perspective," Winnipeg, 1982 (videotape).

8. Pederson, Jean J. and A. Kent. Geometric Playthings to Color, Cut and Fold. San Francisco: Troubador Press, 1973.

9. Schattschneider, Doug and Wallace Walker. M. C. Escher Kaleidocycles, New York: Ballantine, 1977 (layouts).

10. Simmons, Seymour and Marc S.A. Winer. Drawing: The Creative Process. Englewood Cliffs, N.J.: Prentice Hall, 1977.

11. Turner, Harry. Triad: Optical Illusions and How to Design Them. New York: Dover, 1978.

12. Vero, Radu. Understanding Perspective. New York: Van Nostrand, 1980.

Op.XII. STATISTICS (½ module)

Data collection is an involved process and must be started before the unit is begun. The project in part D of this module may be started at the beginning of the unit and students can learn the necessary concepts to complete the project as they go along.

OBJECTIVES	NOTES TO TEACHERS
A. INTRODUCTION	
The student should be able:	
1. to define statistics;	Statistics is a branch of mathematics which deals with gathering, organizing, displaying, and interpreting data.
2. to discuss the role of statistics in modern society;	Inflation rates, interest rates, consumer reports, surveys, and opinion polls are statistics which influence the decisions of individuals and politicians. Organizations, agencies, and corporations produce a mass of statistical data. Other groups interpret the data and may arrive at different conclusions based on the same set of statistics. Ordinary citizens do not have the time or mathematical ability to conduct their own research. They must therefore base their decisions on the results other people have compiled and analyzed. To make meaningful decisions, it is important that they know how the data was compiled, whether the group surveyed was representative, whether the sample was biased, and whether the statistics are being used in a misleading way.
3. to recognize the need to organize raw data.	Students could be given a set of raw data. Example 1. The accompanying marks are from a recent grade 10 mathematics test: 56,63,62,82,75, 59,45,55,65,71, 75,44,80,57,64 66,61,66,78,75

When asked to organize the marks in a meaningful fashion, some students may arrange them in order of magnitude from highest to lowest (called an array) while others may suggest that there are 4 fifties, 2 failures, etc. This latter statement is unacceptable because of the assumption that 50 is a pass mark. Also, a simple frequency distribution table could be drawn:

STUDENT MARKS

Class Boundaries	Frequency
90-99	0
80-89	2
70-79	5
60-69	7
50-59	4
40-49	2
30-39	0
	20

Example 2:

A Student Council wishes to order 250 T-shirts as part of a fund raising drive to send the volleyball team to a provincial competition. T-shirts come in sizes S, M, L, XL.

Students could conduct a noon hour survey of 21 students selected at random. The results of the survey could be written on the board in an unorganized way as S, M, M, L, M, XL, S, M, . . .

The students could organize the raw data in a simple frequency distribution table and then determine how many of each size of T-shirt the council should order.

T-SHIRT SURVEY

Size	Tally	Frequency
XL	//	2
L	~~////~~ /	6
M	~~////~~ ~~////~~	10
S	///	3
		21

OBJECTIVES	NOTES TO TEACHERS
B. SAMPLING The student should be able:	
1. to distinguish between a census and a sampling of the population;	In a census, all members of the population are polled. In a sampling, only a percentage of the total population is surveyed.
2. to use various methods for collecting data;	Data can be collected in various ways. The method used may be a door-to-door survey, a telephone survey, or a mail-in survey. The data may be obtained by measuring (the length of cars in a parking lot) or by counting (the number of absences from school on each day of the week).
3. to describe ways in which the collection of data may be biased.	Discuss ways in which bias may occur. Can the selection of respondents be biased? Can the manner or voice of the interviewer bias the results? What about the wording of questions on a questionnaire? In a random sample, every member of the population must have an equal chance to be selected.
C. ORGANIZING DATA The student must be able:	
1. to calculate the following measures of central tendency or "averages" using ungrouped data: a) the arithmetic mean,	The arithmetic mean is an average obtained by dividing the sum of a group of numbers by the number of numbers. Example: Find the arithmetic mean of the following: 78, 82, 83, 79, 23 Solution: $\frac{78+82+83+79+23}{5} = \frac{345}{5} = 69$

OBJECTIVES

NOTES TO TEACHERS

b) the median,

When an odd number of numbers are arranged in order of size, the middle number is the median.

Example: 23,78,79,82,83. The median is 79.

When the number of numbers is even, the median is the number half way between the two middle numbers.

Example: 2, 6, 7, 8, 15, 24. The median is 7.5.

c) the mode;

The mode is the number occurring most frequently in a set of numbers.

Example: 2,3,3,3,6,7,12,14. The mode is 3.

It should be noted that in sets of continuous data, such as racing times measured to hundredths of a second, the mode may be meaningless since it is unlikely that two racers would have exactly the same time.

2. to select the measure of central tendency most representative of the group of data;

Example:

Find the mean, median, and mode of the following:

2,8,4,9,6,5,4,3,2,8,9,3,7,9

Mean = 5.6 Median = 5.5 Mode = 9

Which is the best measure of central tendency?

Each kind of measure is appropriate for certain types of data. One very high or very low number can distort the mean. The median is not affected by an extremely high or low number but may not be representative either. For example, a group of 5 grandmothers and 4 infants might have a median age of 74, a mean of 42, and a mode of 80.

In a normal distribution, the mean, mode, and median would be identical. Most things, however, are not statistically normal.

Examples:

If a man lays with his head in an oven (40°C) and with his feet in an ice bucket (0°C), could we say that on the average (mean) he is quite comfortable?

OBJECTIVES

3. to determine the range of ungrouped data;

4. to organize sample data using a frequency distribution table;

NOTES TO TEACHERS

Did you know that the employees at Mr. Midas Skinflint's custom auto body earn an "average" of $20 000 a year? What you may not know is that each of his 3 employees has a salary of $7000 while he pays himself $59 000 a year.

Occasionally, measures of central tendency are not meaningful and the extremes must be considered. If engineers were to design highways to carry the "average" traffic, and bridges to support the "average" load, the result would be chaos.

The range is the difference between the largest and the smallest numbers in a set of data.

Example:

Determine the range for the following data:

24, 56, 78, 56, 54, 32, 76, 13, 87, 46, 88, 29

Answer: Range is 88-13 = 75.

The range is only one measure of dispersion. The calculation of deviation and other types of dispersion are left to subsequent grades.

The typing speed in words per minute of students in a grade 10 class were:

39, 18, 26, 34, 39, 44, 39, 29,
33, 34, 38, 45, 45, 41, 48, 56,
42, 46, 48, 43.

OBJECTIVES

NOTES TO TEACHERS

Frequency Distribution Table:

Class Boundaries	Class Mark(x)	Tally	Frequency(f)	(x)(f)
17.5-22.5	20	/	1	20
22.5-27.5	25	/	1	25
27.5-32.5	30	/	1	30
32.5-37.5	35	///	3	105
37.5-42.5	40	~~////~~ /	6	240
42.5-47.5	45	~~////~~	5	225
47.5 -52.5	50	//	2	100
52.5-57.5	55	/	1	55
		Totals	20	800

5. to calculate the mean for grouped data;

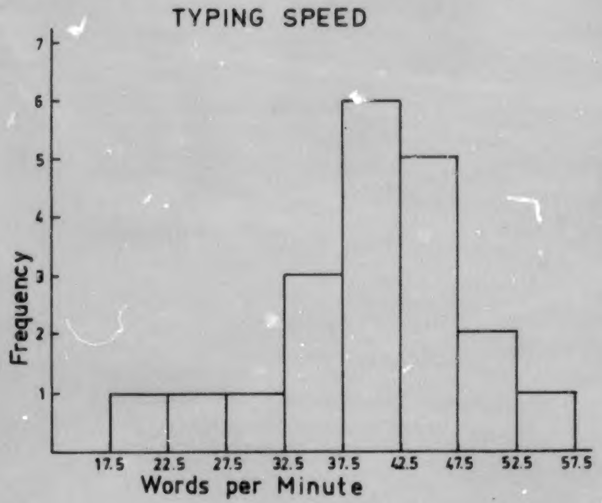
To obtain the mean from data grouped in a frequency distribution table, divide the sum of the numbers in the (x)(f) column by the sum of the numbers in the frequency column.

From the previous table, the mean = $\frac{800}{20}$ = 40 words per minute.

6. to display data using a histogram.

D. PROJECTS

The student should complete a project which involves collecting the data, organizing and/or grouping the data, and displaying the data in a histogram.



Projects may be done by individuals rather than by the class as a group.

Suggestions for projects:

1) Place 100 beans of two different colors in a box. Have a student draw five beans from the box, recording the color of each and then replacing each before drawing the next to ensure that every bean has the same chance of being selected (random sample).

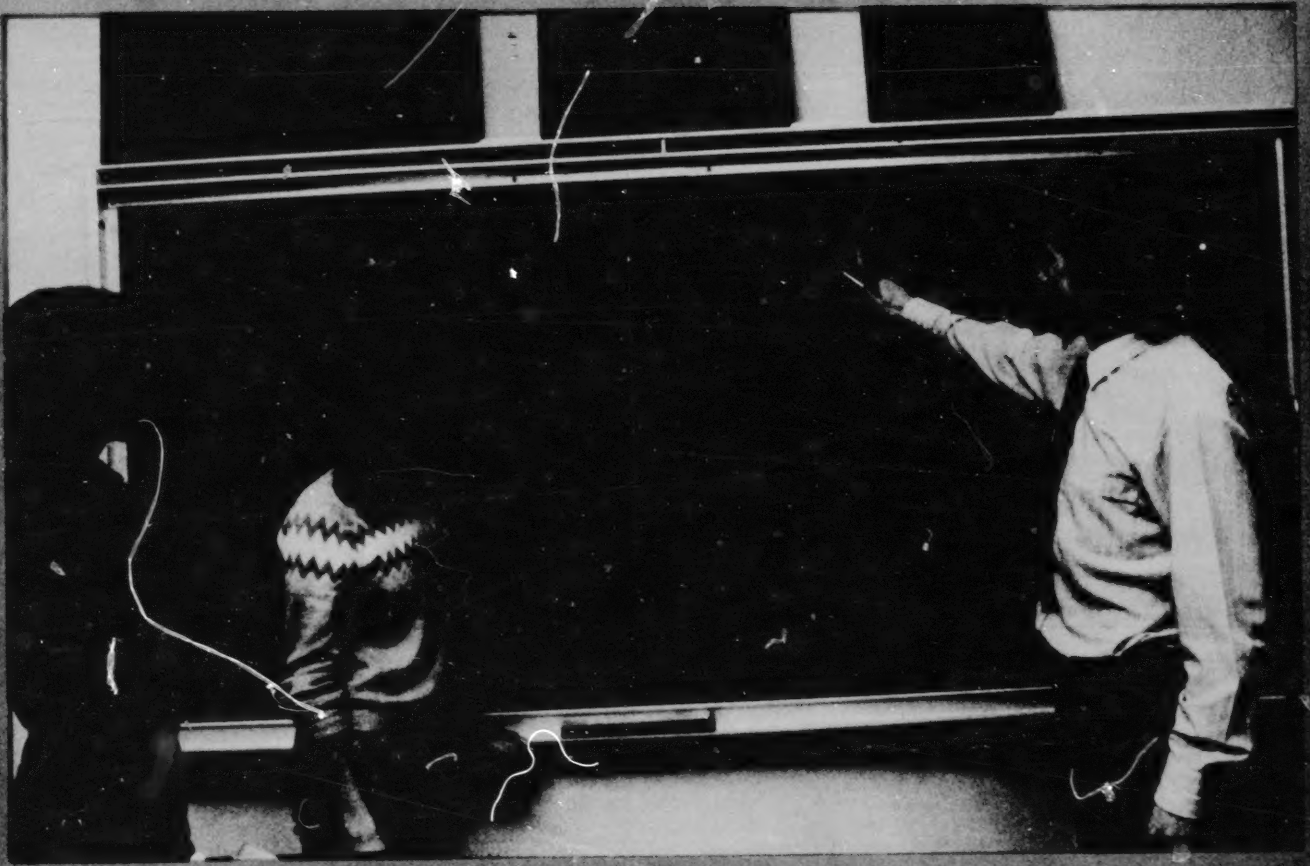
After five beans have been drawn, the number of beans of each color in the box could be estimated.

Samples of ten, twenty and forty beans can be drawn, and the differences in the estimations accompanying each can be noted. A census of the beans can be conducted, the results of which can be compared with the previous estimates.

This could lead to a discussion of public opinion polls.

2) See Op.VIII Graphing for additional project topic suggestions.

mathematics 100



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MATHEMATICS 100

THE 00 MATHEMATICS PROGRAM

Mathematics 100, 200, 300 are designed for students wishing to pursue further studies in mathematics or science-related programs at post-secondary institutions, which makes these the most "vocational" of all mathematics courses in Manitoba. So that the 00 programs will be more meaningful, greater emphasis should be placed on problem solving and practical applications in order to enhance the thinking skills of students.

Mathematics 100, 200 each contain 50% algebra, 40% geometry and 10% consumer mathematics. Mathematics 300 contains 80% core and 20% option topics selected from algebra, analytic geometry and trigonometry.

GENERAL GOALS OF THE 00 PROGRAM

In identifying the goals of the 00 Mathematics curriculum, upon which this guide is based, the present needs of individuals and society have been considered. The student goals that follow are designed to develop:

1. a proficiency in problem solving;
2. an understanding and a mastery of basic algebra skills;
3. the ability to observe and to identify geometric relationships involving two-dimensional figures;
4. the practice of deductive and inductive reasoning;
5. the ability to interpret graphs and to display data graphically;
6. the ability to read mathematics with understanding;
7. the realization that mathematics is important for all consumers;
8. a positive attitude towards, and a curiosity about, mathematics;
9. the capability for post-secondary training in mathematics.

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STRUCTURE OF THE 100 COURSE

Each module accounts for 14 periods (40 minutes each) of new work in a non-semestered system. This should allow approximately 4 periods per module for review, testing, diagnosis or enrichment. Ten modules comprise a full credit. Core modules are compulsory, and Mathematics 100 has the equivalent of 8½ core modules. The remaining equivalent of 1½ option modules are to be selected from geometry topics (½ module) and consumer mathematics (1 module).

Note: Core topics are to be completed to the level specified, bearing in mind the suggested time allotments. Time must be scheduled in order that subjects from each of the option topic areas receive adequate coverage.

The following chart lists the topics identified for the Mathematics 100 course and indicates the allotted time in terms of modules.

Core Topics

Algebra (5 modules)

			Page References
C.I.	Equations	(1 module)	260
C.II.	Polynomial Operations	(1 module)	263
C.III.	Factorization and Rational Expressions	(1½ module)	271
C.IV.	Radicals	(1 module)	275
C.V.	Simultaneous Equations	(½ module)	280

Geometry (3½ modules)

C.VI.	Coordinate Geometry	(½ module)	283
C.VII.	Synthetic Geometry	(3 modules)	286
	Descriptive, Inductive, and Measurement Geometry	(minimum ½ module)	
	Deductive Geometry	(minimum 1½ modules)	

Option Topics

Geometry Option (Choose one of the following)

Op.I.	Vectors	(½ module)	299
Op.II.	Orienteering	(½ module)	303

Consumer Mathematics (Choose one of the following)

Op.III.	Money Management	(1 module)	306
Op.IV.	Consumer Statistics	(1 module)	309

FEATURES OF THE 100 COURSE OUTLINE

1. Although topics are identified as ranging from ½ module to 1½ modules in length, they may be taught in a block or spread throughout the program in an attempt to stimulate both conceptual development and interest.

2. Students are expected to achieve a high level of mastery of the stated objectives for any module. Suggestions, methodology and examples are included to clarify the objectives and expectations.

3. Where times are suggested, they are identified in terms of 40-minute classes and should be considered as an aid in determining the extent/depth to which the content should be studied.

SEQUENCING

Mathematics 100 consists of eleven units in a topical arrangement. A preferred order has not been identified in the guide. Rather, the order to be followed should be determined by the teacher in accordance with the texts that are available. Where content is sequentially out of order in a given unit, topic, or module, reference is made to prerequisite concepts in other modules within the program outline. For example, the study of algebra may or may not begin with a study of C.I Equations. Or C.VI Coordinate Geometry could be studied prior to C.V Simultaneous Equations. Several options are possible.

1. Some teachers may prefer to complete all the algebra or all the geometry before beginning other topics.

2. Other teachers may wish to integrate topics from algebra, geometry, and consumer mathematics.

3. Others may alternate complete modules from algebra, geometry, and consumer mathematics.

4. Still others may choose to intersperse consumer mathematics among the other topics using, for example, one period every week or two.

READING IN MATHEMATICS

Although researchers differ over how students learn mathematics, most agree that students' mathematical development correlates with their ability to read and to verbalize mathematics. For this reason, it is necessary for successful mathematics teachers either to remain, or to become, teachers of reading in mathematics.

Reading should be thought of as a developmental process which must be ongoing throughout the curricula. Reading is not a generalized skill which can be applied in specialized fields, such as mathematics, since reading involves interpretation of particular applications. Research indicates that the reading experiences of students in the content areas determine their growth in reading ability and the development of positive attitudes toward reading and, consequently, learning.

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ALGEBRA (5 modules)

In keeping with the National Council of Teachers of Mathematics recommendation that "problem solving be the focus of school mathematics in the 1980's," one of the major aims of this mathematics program is to encourage problem solving. It should be interspersed throughout each module, rather than dealt with as a separate topic at a module's conclusion. Most modules in the algebra section of this guide begin with an application/problem which could be used to ensure relevancy or to motivate students.

C.I. EQUATIONS (1 module)

A simple application/problem is found in the 7-9 Mathematics (1979) guide:

> A rectangular TV screen is 10 cm wider than it is high. A larger one is 2 cm wider and 3 cm higher and has 240 cm^2 more area. Determine the dimensions of the first screen.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to solve equations of the following types:	
a. linear,	Students should become proficient in solving the following types of equations: 1) $5c + 3 = 11$ 2) $2d + 7 = 5d + 1$ 3) $2(e-3) = 13$ 4) $5(f-5) - 2(f-4) = 4$ 5) $4(g-3) + 5(g-2) = 3(2g+4)$ 6) $\frac{h+3}{4} = \frac{h+5}{6}$ 7) $\frac{k}{3} - \frac{1}{2} = \frac{1}{4}$ 8) $\frac{3m}{2} - \frac{5m}{4} = \frac{3m+1}{6} + 3$ Types 6 to 8 should be considered together in an attempt to show students under which conditions cross-multiplication is acceptable.

OBJECTIVES	NOTES TO TEACHERS
b. equations which can be reduced to linear form having numerical coefficients,	Example: Solve for W in each of the following: 1) $\frac{2}{W} = \frac{3}{4}$ 2) $\frac{2}{W} = \frac{3}{W + 15}$
c. literal equations,	Solving literal equations is important in manipulating formulae and in isolating the operations in the solution of an equation. Students should be able to solve for any variable in equations of the type $\frac{ax + b}{c} - d = f$ where $c \neq 0$. More complicated types of literal equations should be solved after completion of C.III Factorization and Rational Expressions. Examples could include solving: 1) $A = P + PRT$ for "P" (Note: For some values of A, R, and T this equation has no solution. However, this point need not be stressed during this module.) 2) $\frac{1}{p} = \frac{1}{q} - \frac{1}{t}$ for "t"
d. simple quadratic equations;	This topic could be studied in conjunction with C.IV Radicals. Students are expected to solve for "x" in equations of the form $ax^2 + p = t$ where a, p and t are integers. Students must realize that in solving equations of the type $x^2 = k$ where $k > 0$ there are two solutions; i.e., $x = \sqrt{k}$ or $x = -\sqrt{k}$. Literal equations or formulae such as $E = \frac{1}{2}mv^2$ (solve for v) should be considered here.

OBJECTIVES	NOTES TO TEACHERS
2. to evaluate formulae by substitution;	The evaluation of formulae should be considered in conjunction with literal equations. Example: The velocity of a satellite orbiting the earth at the equator is given by $v = \frac{2\pi(3960 + h)}{p}$ where v is the velocity in km/hr., p is the time in hours for one orbit, and h is the height of the satellite above the earth in kilometres. Find the velocity in kilometres per hour if the orbit time is 24 hr., the height is 30 000 km and $\pi \doteq 3.14$.
3. to reduce equations to the quadratic form $ax^2 + bx + c = 0$ where a, b, c are integers and $a \neq 0$;	This should be studied after the work on rational expressions in C.III is covered. Example: Convert $\frac{2}{x + 1} = 3 + \frac{4}{x - 2}$, $x \neq -1, 2$, into the form $ax^2 + bx + c = 0$ where a, b, c are integers and $a \neq 0$. Note: The important thing here is that students read the question and follow the directions. Teachers should emphasize reading in mathematics and should encourage students to do only what is required in each question. Questions should not be so typical that students think they know what to do before they examine the exercise.
4. to translate word phrases into algebraic expressions;	Review as necessary.
5. to solve problems which involve translation into an equation.	It should be noted that problem solving is an on-going process, and that a great deal of time is required before students can develop and appreciate the techniques involved. Discourage rote memorization of 'type' problems. Test questions should involve those not shown to students in class but which involve the same solution techniques. An attempt should be made to use relevant applications/problems. There is a multitude of problems including: rate, time, distance, finance, work, plane geometric figures, etc., which could be used. Reference: Foerster, Paul A. *Algebra and Trigonometry: Functions and Application.* Menlo Park, Calif.: Addison-Wesley, 1980.

C.II. POLYNOMIAL OPERATIONS (1 module)

Teachers are reminded to examine the appropriate objectives for grades 7-9 mathematics which appear at the beginning of this guide. Algebraic operations are de-emphasized in these grades, and it cannot be assumed that all students have taken the algebra option listed in the grade 9 outline.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to identify and to classify algebraic terms;	Definitions and technical terminology should not be over-emphasized. The skill of manipulating terms is of far greater importance. Students should be shown the construction of a term: sign-coefficient-variable(s). Care should be taken to point out that $a = 1a^1$ (coefficient and exponent of 1 understood).
2. to combine like terms;	Questions should be arranged both horizontally and vertically. Students should realize that variable(s) do not change when terms are combined; in fact, the process involved is just coefficient combination. Examples: 1) Add: $\begin{array}{r} 4a - 5b + 6c \\ \underline{-7a + 8b - 9c} \end{array}$ 2) Combine like terms: $14u - 17v + 20w - 23u + 26v - 29w$ The arithmetic skill being practiced is the combination of signed numbers: $\begin{array}{r} +10 \\ \underline{+10} \\ +20 \end{array} \quad \begin{array}{r} -42 \\ \underline{-58} \\ -100 \end{array}$ Rule: Add and place the prevailing sign $\begin{array}{r} +20 \\ \underline{-18} \\ +2 \end{array} \quad \begin{array}{r} -34 \\ \underline{+16} \\ -18 \end{array}$ Rule: Subtract and place the 'dominant' sign

OBJECTIVES

3. to perform algebraic subtraction;

4. to use the basic exponent laws as applied to terms;

NOTES TO TEACHERS

Students must realize that $10 - 12 = 10 + (-12)$ and that $14 - (-6) = 14 + 6$ in order to perform algebraic subtraction.

Examples:

1) $a + 2b - 3c - (-4a - 5b + 6c)$. In such examples, one approach is to remove the brackets using multiplication and then to combine like terms.

2) Subtract:
$$\begin{array}{r} a + 2b - 3c \\ \underline{-4a - 5b + 6c} \end{array}$$

Numerical coefficients should involve both integral and rational values (including decimal fractions).

> Note: Long periods of time should not be devoted to a theoretical study of exponents and the so-called laws of exponents. Students should be able to handle exponents in their algebraic simplification exercises. Negative and fractional exponents need not be discussed at this time as they will be dealt with in Mathematics 300.

The multiplication of two terms can be considered as a fusing together to form a new term.

1) $(-6a^2b)\ (-2a^3b^2)$ can be expanded as:

$[-6(a)(a)(b)]\ [-2(a)(a)(a)(b)(b)] = 12a^5b^3$ by actual counting of letters. It is expected that students will quickly formulate the "adding exponents" shortcut.

2) $\dfrac{-12a^4c^3}{24ac^5}$ = can be expanded as:

$$\frac{-12(a)(a)(a)(a)(c)(c)(c)}{24(a)(c)(c)(c)(c)(c)}$$

$$= \frac{-1a^3}{2c^2} \text{ or } \frac{-a^3}{2c^2} \text{ or } -\frac{a^3}{2c^2}$$

Note: Negative exponents need not be discussed at this time. Again, the "subtract exponents" rule should be easily discerned by students.

OBJECTIVES

NOTES TO TEACHERS

3) $(-3x^3y^4w^5)^2$

This can be done as:

$(-3x^3y^4w^5)(-3x^3y^4w^5) = +9x^6y^8w^{10}$

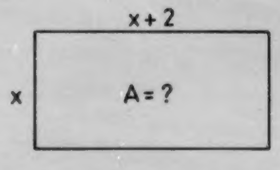
using the ideas in the previous example(s).

To reinforce arithmetic skills, both integral and rational coefficients should be used.

5. to perform general algebraic multiplication;

Teachers could use geometric examples to introduce algebraic multiplication.

Example:



Determine the area of this figure using the given dimensions. (For simplicity, units have been omitted.)

The concept of a polynomial as a string of terms separated by + or - signs must be stressed. It should be pointed out that there are two special polynomials, binomials and trinomials (viz, K + M and K + M + N where K, M, N are arbitrary terms). Types of questions should include:

1) 1 term by 1 term (1 x 1)

$(-2a^2b^4)(4ab^3) = -8a^3b^7$

Students must realize that 1 x 1 multiplication is the key to all other algebraic multiplications.

2) 1 term by 2 terms (1 x 2)

$2a(4a + 7b) = 8a^2 + 14ab$

OBJECTIVES	NOTES TO TEACHERS

In the 1 x 2 case, a problem could be posed:

> Determine possible dimensions for a TV screen whose perimeter has been given as 120 cm.

Students should deduce that since $A = (L)(W)$ and $L + W = 60$, then $W = 60 - L$ and $A = L(60 - L)$. Then the question of what dimensions will maximize viewing area can be posed.

3) 2 terms by 2 terms (2 x 2)

$(2a + 3b)(4c - 5d) = 8ac - 10ad + 12bc - 15bd$

Middle terms do not always combine.

$$(5a + 7b)(3a - 2b) = 15a^2 - 10ab + 21\ ab - 14b^2$$

$$= 15a^2 + 11ab - 14b^2$$

Simplification has occurred by combining the middle terms. Students must realize that "ab" is identical to "ba."

> _Note:_ Multiplication should be extended to include all possible products of "m" terms by "n" terms.

4) squaring

$$(3k - 5)^2 = (3k - 5)(3k - 5)$$

$$= 9k^2 - 30k + 25$$

After a few examples, encourage "automatic" squaring: square + double product + square.

Example:

$$(2a + 3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2$$

$$= 4a^2 + 12ab + 9b^2$$

OBJECTIVES | **NOTES TO TEACHERS**

5) multiplying to obtain a difference of squares

$(6m + 7n)(6m - 7n) = 36m^2 - 49n^2$

Ask students why the middle term(s) do not appear.

Then, reverse the process and have students produce two expressions whose product is $64b^2 - 81c^2$.

6. to add or to subtract products of binomials;

Examples:

1) addition

$(5x + 2)(2x - 1) + (4x - 3)^2$

2) subtraction

$(5x - 3)^2 - (2x - 7)^2$

$(2x - 3y)^2 - (5x + y)^2$

7. to use Pascal's triangle in binomial expansions;

Pascal's triangle is suggested as a means for determining coefficients in binomial expansions, such as $(A + B)^n$ where "n" is a whole number.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
. . .

Pascal's triangle should be related to the following expansions:

$(A + B)^0 = 1$ (An explanation of the zero exponent may be necessary here.)

$(A + B)^1 = 1A + 1B$

$(A + B)^2 = 1A^2 + 2AB + 1B^2$

$(A + B)^3 = 1A^3 + 3A^2B + 3AB^2 + 1B^3$

etc.

Many algebraic skills and applications can be practiced using the patterns in Pascal's triangle.

Examples:

1) Expand and simplify $(2a + 5b)^4$

Solution:

$(2a + 5b)^4 = (A + B)^4$ where $A = 2a$ and $B = 5b$

$(A + B)^4 = 1A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + 1B^4$ so

$$(2a + 5b)^4 = 1(2a)^4 + 4(2a)^3(5b) + 6(2a)^2(5b)^2 + 4(2a)(5b)^3 + 1(5b)^4$$

$$= 16a^4 + 160a^3b + 600a^2b^2 + 1000ab^3 + 625b^4$$

2) Expand and simplify $(3k - 4n)^3$

Solution:

$(3k - 4n)^3 = (A + B)^3$ where $A = 3k$ and $B = -4n$ so that

$$(3k - 4n)^3 = (3k)^3 + 3(3k)^2(-4n) + 3(3k)(-4n)^2 + (-4n)^3$$

$$= 27k^3 - 108k^2n + 144kn^2 - 64n^3$$

OBJECTIVES | NOTES TO TEACHERS

3) Any event which is duplicated in succession and has two outcomes is modelled after the numerical configuration in Pascal's triangle. Teachers may want to indicate how the expansions are used in conjunction with probability [e.g., coin tossing and determining H (heads) or T (tails)]. Example: If a coin is tossed 3 times, what is the probability of obtaining 1H and 2T's?

8 possible outcomes:
HHH
HHT
HTH
* HTT
THH
* THT
* TTH
TTT

The three desirable outcomes are indicated by asterisks so the probability is 3/8.

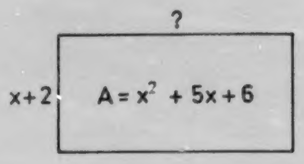
$$(A + B)^3 = 1A^3 + 3A^2B + \underline{3AB^2} + B^3$$

The $3AB^2$ represents the event of 1H and 2T's and the coefficient of 3 represents the outcomes that count. The sum of the coefficients is 8 and hence the probability $= \frac{3}{8}$.

8. to perform general algebraic division;

Teachers could include some geometric motivation for division.

Example:



Determine the missing dimension of this figure. (For simplicity, units have been omitted.)

OBJECTIVES

NOTES TO TEACHERS

Examples should include the following types:

1) Divide $-45m^3n^4p^5$ by $9m^5n^4p^3$.

2) Divide $6x^2 + 3x$ by $3x$.

Such examples can be expressed as:

$$\begin{array}{r r}
 & 2x + 1 \\
3x\big) & \overline{6x^2 + 3x} \\
 & \underline{6x^2 } \\
 & 3x \\
 & \underline{3x} \\
 & 0
\end{array} \text{ (remainder)}$$

3) Divide $2a^2 - 5ab - 3b^2$ by $a - 3b$ (Answer has no remainder.)

4) Divide $m - 3n$ into $2m^2 - 4mn - 3n^2$ (Answer has a remainder.)

5) Divide $27k^3 + 64t^3$ by $3k + 4t$ (Solution involves missing terms.)

9. to express the result of an algebraic division in alternate forms.

It should be pointed out that there are alternate forms for expressing the result of division question.

Example:

Divide $3a^3 + 2a^2 + a + 11$ by $a + 1$

$$\begin{array}{r r}
 & 3a^2 - a + 2 \\
a+1\big) & \overline{3a^3 + 2a^2 + a + 11} \\
 & \underline{3a^3 + 3a^2} \\
 & -a^2 + a \\
 & \underline{-a^2 - a} \\
 & 2a + 11 \\
 & \underline{2a + 2} \\
 & 9
\end{array}$$

The result of the division can be expressed as follows:

$$3a^3 + 2a^2 + a + 11 = (3a^2 - a + 2)(a + 1) + 9$$

$$\text{Or: } \frac{3a^3 + 2a^2 + a + 11}{a + 1} = 3a^2 - a + 2 + \frac{9}{a + 1}$$

C.III. FACTORIZATION AND RATIONAL EXPRESSIONS (1½ modules)

Factorization and Rational Expressions should be taught as an integrated unit with the rational expressions as an application of the factoring techniques. Teachers should provide relevant examples employing the concepts outlined below.

Consider the following introductory problem:

> A tourist travelled 200 km by car and 240 km by bus. If the tourist went 10 km/h faster by bus than by car, what was the speed of the car if the trip required 8 hours?
>
> (Use $\frac{200}{x} + \frac{240}{x + 10} = 8$, where x is the speed of the car.)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to demonstrate an understanding of the term factoring;	Introduce students to factoring as the reverse process of multiplication.
2. to reduce a rational expression with factored numerator and denominator;	Include examples such as: $\frac{(3x-1)(4a+b)(3x-y)}{(4a+b)(3x-1)(3x-y)}$ or $\frac{(5t-s)(a-4)}{4(5t-s)}$. Students are expected to simplify by reducing the expression. Note: In all examples with rational expressions, non-zero denominators are assumed.
3. to reduce a rational expression using common factoring;	Encourage students first to factor the expression and then to multiply as a check. Common factoring: $2x + 4y = 2(x+2y)$ Reducing rational expression: $\frac{2}{2x + 4y} = \frac{2(1)}{2(x + 2y)} = \frac{1}{x + 2y}$ Note: Time should be spent factoring out -1. This technique is necessary in some reduction exercises. Common factoring: $y - x = -1(-y+x)$ or $-1(x-y)$ Reducing rational expression: $\frac{(a-b)(a+b)}{b-a} = \frac{(a-b)(a+b)}{-1(a-b)} = \frac{a+b}{-1}$ or $-(a+b)$

Note: Applications of different types of factoring have been included to show a use for factoring and could be considered as enrichment, time permitting.

Problem Applications of Common Factoring

Example 1: Determine the accumulated amount of \$100 compounded at 10% per annum for 5 years.

Solution(s):

Accumulated amount at the end of:

Year 1 is \$100 + \$10 (int.) = \$110
Year 2 is \$110 + \$11 (int.) = \$121
Year 3 is \$121 + \$12.10 (int.) = \$133.10
Year 4 is \$133.10 + \$13.31 (int.) = \$146.41
Year 5 is \$146.41 + \$14.64 (int.) = \$161.05

Using factoring, the formula $S = P(1 + r)^n$, where S = accumulated amount, P = principal, r = rate of interest per annum, and n = number of interest periods, can be derived as follows:

Accumulated amount at the end of:

Year 1 is $P+Pr = P(1+r)$
Year 2 is $(P+Pr) + (P+Pr)r = (P+Pr)(1+r)$
$= P(1+r)(1+r)$
$= P(1+r)^2$

. . .

Year 5 is $P(1 + r)^5$

After n years the accumulated amount could be determined using $S = P(1 + r)^n$

$S = 100(1 + 0.10)^5 = 100(1.1)^5 = 100(1.61051) = 161.051$

Therefore, after 5 years the accumulated amount is \$161.05.

Example 2: The number of diagonals in an n-sided polygon can be determined using $\frac{n^2 - 3n}{2}$ or $\frac{n(n - 3)}{2}$. It can be shown inductively that this formula is true.

OBJECTIVES	NOTES TO TEACHERS
4. to factor a difference of squares;	Students should be exposed to the following types of examples. 1) Factor: (a) $x^2 - 4y^2$ (b) $4x^2 - 0.09b^2$ (c) $a^4 - b^4$ 2) Factor and simplify: $(2x+1)^2 - (5x-3)^2$ Also, students should realize that $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a-b$. Normally such "factoring" is not advantageous to the simplification process.
5. to reduce a rational expression using difference of squares and/or common factoring techniques;	Example: Reduce: $\frac{x^2 - 49y^2}{2x^2 + 14xy}$ Problem Applications of the Difference of Squares Example: The area of an annulus (ring or washer) is given by $A = \pi R^2 - \pi r^2$ or $\pi(R-r)(R+r)$.
6. to factor trinomials;	Exercises should include the forms $x^2 + bx + c$ and $ax^2 + bx + c$ where a, b, c are integers.
7. to reduce rational expressions using common, difference of squares, and/or trinomial factoring techniques;	Examples: Reduce: 1) $\frac{x^2 + 6x + 8}{2x^2-8}$ 2) $\frac{x^2 - 9y^2}{x^2 - 5xy - 24y^2}$ 3) $\frac{a^2 - 2a - 3}{2a^2 - a - 15}$

Note: Factoring of sum and difference of cubes will be dealt with in Mathematics 300.

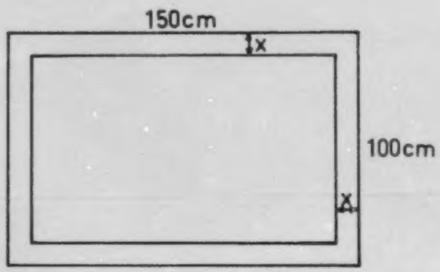
OBJECTIVES

NOTES TO TEACHERS

Problem Applications Using Trinomials

Example:

Weather stripping is placed around a window. If the window measures 100 cm by 150 cm and the area of the weather stripping used is 984 cm^2, how wide is the weather stripping?



Let

x = width of weather stripping
A = area of window not covered by weather stripping

A is $(150-2x)(100-2x) = (150)(100) - 984$

$x^2 - 125x + 246 = 0$

$(x-2)(x-123) = 0$

$x = 2$ or 123

The weather stripping is 2 cm wide.

Note that solving quadratic equations by factoring trinomials is not part of this course outline. This example is included simply to show a use for trinomial factoring.

8. to multiply and to divide rational expressions;

9. to add and to subtract rational expressions.

This objective could be integrated with factoring as was done with reducing rational expressions.

Exercises should include simplifying the following types of expressions:

1) $\frac{3x + 1}{2} - \frac{x + 1}{3}$

2) $\frac{3}{x + 1} + \frac{4}{x - 1}$

3) $\frac{2x}{x^2 + 5xy} - \frac{x}{x^2 + 8xy + 15y^2}$

Note: Undue time should not be spent on the addition/subtraction of rational expressions.

C.IV. RADICALS (1 module)

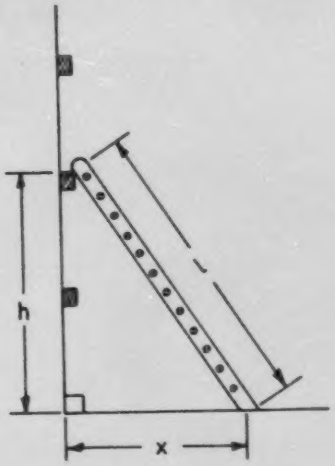
Traditionally in Manitoba, radicals have been introduced in conjunction with exponents. But this guide recommends that the topic of exponents be integrated with the section on polynomials covered here in C.II. Work on negative and fractional exponents is not part of this course outline but rather will be included when necessary in succeeding grades. Teachers are reminded that excessive time should not be spent reviewing exponents prior to studying radicals and that complicated examples involving exponents should not be considered.

The following problems could be used to introduce the need for radicals.

Problem 1

Restorers are to remove a priceless stained-glass window from a cathedral for repair and cleaning. To do this, a 10 m ladder must be set against the ledge below the window. The base of the ladder must be placed at the correct distance from the wall or the tip of the ladder may smash either the window to be repaired or the one below. How far from the wall must the base be placed if the ledge is a) 8 m up? b) 7 m up?

Consider the following diagram as an aid in solving the problem.



Students will need to use the Pythagorean Relation to solve the problem.

The answer for a) is found as follows:

$$h^2 + x^2 = \ell^2$$

$$8^2 + x^2 = 10^2$$

$$x^2 = 36$$

$$x = \pm 6$$

The answer is 6 m since distance is positive.

The b) part of the problem involves a radical,

$$x^2 = 51$$

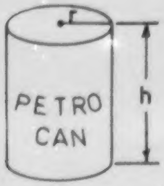
$$x = \pm\sqrt{51}$$

The answer is $\sqrt{51}$ m.

Problem 2

A manufacturer wants to make his 1 L oil cans 15 cm high. What must be the radius of the cans?

This applied problem relies on students' ability to work in terms of volume. Students may have to be guided through the initial work, which follows:



Volume = (Area of base)(height)

$V = (\pi r^2)(h)$

$V = (\pi r^2)(15)$

$V = 15\pi r^2$ in this case and $\frac{V}{15\pi} = r^2$

Hence $r = \sqrt{\frac{V}{15\pi}}$

If $\pi \doteq 3.14$ the problem can be solved easily using a calculator.

$r \doteq \sqrt{\frac{1000}{(15)(3.14)}}$ (Note that 1 L = 1000 cm^3.)

$r \doteq 4.6$

∴ the radius is 4.6 cm, correct to 1 decimal place

OBJECTIVES

The student should be able:

1. to find square roots of non-negative rational numbers using either a calculator or a table;

NOTES TO TEACHERS

Teachers may wish to review the real number system at this time.

The concept of square root should be taught carefully. The principal square root of 9, by agreement, is $\sqrt{9} = 3$ and not ± 3. However, students must realize that $-\sqrt{9} = -3$.

Note: The square root algorithm may be taught as enrichment, time permitting.

OBJECTIVES

2. to demonstrate an understanding of the Pythagorean Relation by solving application problems;
3. to simplify algebraic square roots;
4. to perform basic operations on terms involving radicals;

NOTES TO TEACHERS

Teachers may wish to derive/prove the Pythagorean Relation.

By definition $\sqrt{x^2} = |x|$, which means that if x is negative, $\sqrt{x^2} \neq x$. For example, $\sqrt{(-3)^2} = \sqrt{9} = 3$. Because of this, $\sqrt{50x^2} = 5|x|\sqrt{2}$ or $\doteq 7.071\ |x|$ and $\sqrt{x^2y^5} = |x|y^2\sqrt{y}$, if $y \geq 0$. This definition should be pointed out to students but need not be emphasized at this stage.

For the remainder of this module it is assumed that x and y are positive numbers.

Examples could include:

1) $\sqrt{50x^2} = 5x\sqrt{2}$ or $\doteq 7.07x$

2) $\sqrt{x^2y^5} = xy^2\sqrt{y}$

Examples:

1) $2\sqrt{x} + 3\sqrt{x} = 5\sqrt{x}$

2) $2\sqrt{x^5} - 3\sqrt{x} = 2x^2\sqrt{x} - 3\sqrt{x}$

3) $(3\sqrt{x})(5\sqrt{y}) = 15\sqrt{xy}$

4) $(9\sqrt{2x})(\sqrt{3x^5}+2) = 9x^3\sqrt{6} + 18\sqrt{2x}$ or $\doteq 22.05x^3 + 25.46\sqrt{x}$

5) $(\sqrt{x}+\sqrt{5})(3\sqrt{x}-2\sqrt{5}) = 3x+\sqrt{5x}-10$

6) $(2\sqrt{x}-1)^2 = 4x-4\sqrt{x}+1$

7) $(3\sqrt{x}+\sqrt{2})(3\sqrt{x}-\sqrt{2}) = 9x-2$

8) $\dfrac{12\sqrt{3x}}{3\sqrt{x}} = 4\sqrt{3}$

OBJECTIVES

5. to rationalize the numerator or the denominator of a radical expression;

NOTES TO TEACHERS

Teachers are encouraged to de-emphasize rationalizing the denominator in numerical expressions. Using calculators, students can approximate $\frac{1}{\sqrt{2}}$, without having to rationalize the denominator.

However, students should be able to convert expressions such as $\frac{1}{\sqrt{x}}$ to $\frac{\sqrt{x}}{x}$ and $\frac{\sqrt{x}}{2}$ to $\frac{x}{2\sqrt{x}}$.

The process of rationalization of a numerator has been included since it is useful in the evaluation of certain limits.

Note: The instruction "rationalize the following expression(s)" should not be used. Instructions should read "rationalize the numerator..." or "rationalize the denominator...."

Examples:

1) Rationalize the denominator in each of the following:

   a) $\frac{x + 1}{\sqrt{x}}$

   b) $\frac{\sqrt{x} - 1}{4 + \sqrt{x}}$

2) Rationalize the numerator of $\frac{\sqrt{x} - 1}{4 + \sqrt{x}}$

OBJECTIVES

6. to solve equations involving radicals.

NOTES TO TEACHERS

Examples:

Solve for x in each of the following:

1) $\sqrt{x} = 4$

2) $2\sqrt{x} = 4$

3) $\sqrt{x-2} + 3 = 5$

4) $\sqrt{x+2} + 3 = 1$ This example reduces to "$\sqrt{x+2} = -2$" which yields no solution. Care should be taken to avoid radical equations which lead to the solving of a general quadratic.

Note: Equations involving two radicals will be introduced as new work in Mathematics 200.

C.V. SIMULTANEOUS EQUATIONS (½ module)

Simultaneous equations can be used to solve a variety of numerical problems. These may deal with telephone and telegraph rates, mixtures, coins, ages, levers, finance, plane geometric figures, or temperature conversions.

Sample Problems:

1) With a tail wind a plane took 4 hours to fly 1920 km. Flying back against the same wind with the same air speed the plane took 1 hour longer. Determine the plane's air speed and the wind speed.

2) A large truck was hauling a load of turkeys and pigs to market. The man at the local weigh scale noticed that the truck contained 86 eyes and 122 feet. How many turkeys and pigs were in the truck? (Possible answers could include: 25 turkeys and 18 pigs, or 1 driver, 24 turkeys, 18 pigs.)

Such problems can be translated to systems of two first degree equations in two unknowns (variables) and then solved.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to determine graphically whether a pair of simultaneous first degree equations are inconsistent (parallel lines), consistent and dependent (the same line), or consistent and independent (lines that intersect in exactly one point);	A review of graphing straight lines is needed. A system of two simultaneous first degree equations is said to be consistent if it has at least one solution and is said to be inconsistent if it has no solution. A system of two simultaneous first degree equations is said to be dependent if one is a non-zero multiple of the other and otherwise is said to be independent. These terms are particularly useful in describing a system of more than two first degree equations. Since graphical solutions tend to be inaccurate, algebraic solutions are necessary.
2. to solve a system of two first degree equations in two unknowns using the addition-subtraction method;	Examples involving rational coefficients should be included in an attempt to encourage arithmetic proficiency. Note: The substitution method for solving a system of two first degree equations in two unknowns will be dealt with in Math 200.

OBJECTIVES

3. to translate problems into pairs of simultaneous first degree equations.

NOTES TO TEACHERS

Students should be required to define each variable used.

Examples:

In each of the following problems, state specifically what each letter/variable represents and then write the two equations which would be necessary to solve the problems algebraically. Do not solve.

1) An insurance company agrees to purchase a number of microcomputers at \$900 each with a monthly service fee of \$15 per machine. If the company pays \$18 360 and if the length of the agreement in months exceeds the number of micros by 4, determine the number of micros purchased and the length of the agreement.

   Solution:

   Let the number of micros be "M" and the length of the agreement in months be "t".

$$900M + 15t = 18\ 360$$
$$t - m = 4$$

2) The cost of admission to a high school basketball game is 90¢ for adults and 50¢ for students. If the gate receipts from 300 paid admissions is \$194.00, how many adults and how many students attended the game?

   Solution:

   Let the number of adults be "x" and the number of students be "y".

$$x + y = 300$$
$$90x + 50y = 19\ 400 \quad \text{or} \quad 0.90x + 0.50y = 194$$

Carrying problems through to their final solution is a time-consuming process. Emphasis should be placed on providing an accurate description of the variables and on establishing the equations rather than on obtaining numerical solutions.

Note: Teachers may wish to introduce linear programming in two variables or the solution of a system of three first degree equations in three unknowns as enrichment topics.

GEOMETRY (4 modules)

The geometry portion of Mathematics 100 is divided according to the following time allotments:

Coordinate Geometry	½ module
Synthetic Geometry	3 modules
Descriptive, Inductive and Measurement Geometry	minimum ½ module
Deductive Geometry	minimum 1½ modules
Geometry Option (Do one of the following)	
Vectors	½ module
Orienteering	½ module

Since it is difficult to completely separate descriptive and deductive geometry, this guide includes sections in which both are integrated.

During the past several decades, the geometry curriculum in Manitoba high schools has placed a strong emphasis on correct terminology and on the formal axiomatic structures of geometry. This almost exclusive emphasis on the axiomatic process has led some students to dislike geometry and has limited their ability to view the subject creatively and to enjoy geometric activity.

This curriculum proposes a blended approach to the study of geometry in Mathematics 100. This approach is designed to build on the intuitive, informal study of geometry begun in previous grades; to introduce the transformational approach to graphing straight lines in coordinate geometry; to integrate deductive and inductive or descriptive geometries; and to allow teachers some flexibility in their approach to geometry in order to maintain student interest.

Sequencing in Geometry

The order outlined in the geometry section of this guide does not match the order found in all existing textbooks. Again, teachers are advised to determine an order in accordance with those texts which are available. On the other hand, it is fair to indicate that pilot teachers have found the outlined order for geometry to be quite workable and satisfactory, particularly when using Geometry: Step by Step (Silverberg, Korytowski and Fast).

Time for Geometry

Available time for geometry has been reduced by 10% to 15% over the previous outline. This time reduction is designed to better reflect classroom practice and to allow for the introduction of consumer mathematics. Sufficient time has not been provided to derive geometric properties on a postulate-deductive base with all students. And in order to complete the program in the available time, several properties will need to be accepted on the basis of manipulation and measurement. Most of the available time is for solving problems and for writing elementary proofs using certain basic geometric properties or facts.

Notation for Segments and Angles

Some books employ AB while others use $\overline{AB}$ to represent the length/measure of segment AB. Similarly, some employ $\angle COB$ while others use $m\angle COB$ to represent the measure of angle COB. Teachers need not use the notation outlined in this guide but are advised to use one notation consistently. It is suggested that the notation used be the same as that found in the students' text(s).

C.VI. COORDINATE GEOMETRY (½ module)

OBJECTIVES

The student should be able:

1. to demonstrate an understanding of the terminology associated with the coordinate plane;
2. to locate and to plot points in the four quadrants and on the axes;
3. to graph a straight line given its equation in the form y = mx + b where "m" and "b" are real numbers;

NOTES TO TEACHERS

Students have worked with the coordinate plane in grades 7-9 and should be familiar with the following terms:

1) coordinate plane,
2) coordinate axes,
3) coordinates or ordered pairs,
4) quadrants,
5) origin.

Review as necessary.

Students should not be encouraged to use only a table-of-values approach to graphing. Graphing using the transformational approach is to be introduced here and will be continued in successive years. This approach to graphing results in a saving of time over traditional methods. It will be useful for determining the equations of specific straight lines in Mathematics 200.

Transformational Approach

Step 1: Begin with the basic graph of $y = x$, where the slope is 1 and the line is at an angle of 45° to the positive x - axis. Proceed to graph $y = 2x$, $y = 3x$, $y = 4x, \ldots$ and $y = 0.9x$, $y = 0.8x$, $y = 0.7x$, ... and to notice the effect which the numerical coefficient "m" has on the line. Generalize to the cases where $m = 0$ (horizontal line) and m is negative. Identify slope as the ratio of the change in y to the change in x.

$$\text{Slope} = m = \frac{\text{change in } y \text{ coordinates}}{\text{change in } x \text{ coordinates}} = \frac{\Delta y}{\Delta x} \text{ or } \frac{\text{rise}}{\text{run}}$$

Step 2: Again, begin with the basic graph of $y = x$. Proceed to graph $y = x + 1$, $y = x + 2$, $y = x + 3$, ... and $y = x - 1$, $y = x - 2$, $y = x - 3$, ... and to notice the effect which the numerical constant "b" has on the line. Identify the y- intercept.

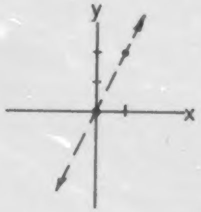
Step 3: Combine steps 1 and 2 in graphing equations of the form $y = mx + b$.

Students quickly learn to sketch the graph of straight lines using this method.

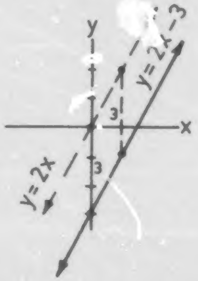
Example: Draw the graph of $y = 2x - 3$.

Solution:

Visualize or graph $y = 2x$.



From $y = 2x$ graph $y = 2x - 3$ by "dropping" the line 3 units.



Note: Encourage students to plot the points necessary for graphing in Steps 1-3 without the aid of a table-of-values.

OBJECTIVES

4. to define the slope and the y- intercept of a line;

5. to determine the value of the slope and the y-intercept:

   a. from the equation of the line in the form $y = mx + b$,
   b. from the graph;

6. to write the equation of a straight line in the form $y = mx + b$:

   a. given the slope and y- intercept,
   b. given the graph from which the slope and y- intercept can be obtained;

7. to write the equation of a vertical line in the form $x = k$, given the graph.

NOTES TO TEACHERS

Students must realize that there are several forms for the equation of a straight line and should be prepared to convert from the forms $Ax + By = C$ or $Ax + By + C = 0$ into the form $y = mx + b$.

For horizontal lines with equations in the form $y = b$, students should realize that $m = 0$ which causes the mx term to disappear.

The slope of a vertical line is undefined and therefore the equation of a vertical line cannot be written in the form $y = mx + b$. The equation of a vertical line has the form $x = k$ where k is a real constant.

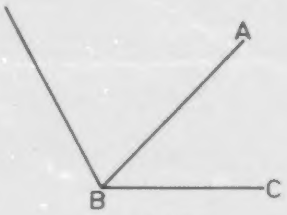
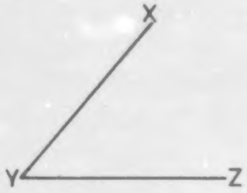
C.VII. SYNTHETIC GEOMETRY (3 modules)

Note: Any reference to "periods" refers to 40-minute periods.

A. POINTS, LINES, PLANES AND ANGLES (4 periods)

Many of the topics in this section review core material from grade 9 mathematics, so little time should be required in covering the terminology and definitions.

This section may be taught at the beginning of the geometry or it may be integrated with other sections as needed. It has been included to establish an intuitive base on which to develop a deductive approach.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to name a segment or angle using standard notation;	Students should realize when a single letter can be used and when three letters must be used to denote an angle.  $\angle ABC$  $\angle Y$ or $\angle XYZ$
2. to estimate and to measure: a. segments to the nearest millimetre or centimetre,	One general goal of mathematics is to encourage students to make reasonable estimations. It is not enough to say "estimate." Teachers must make systematic and regular reference to the skill of estimation. Students should estimate, record the estimate, measure, record the measure and then compare the estimate with the measure in physical situations.

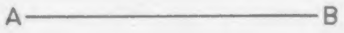
OBJECTIVES

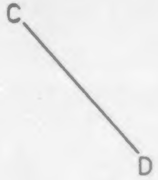
b. angles to the nearest degree;

NOTES TO TEACHERS

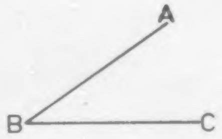
Examples:

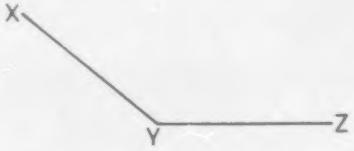
1) Estimate the lengths of segments AB and CD to the nearest millimetre.





2) Estimate the measures of ∠ABC and ∠XYZ.





3) Draw a reasonable approximation of the following without using a ruler or protractor:

a) a segment 3.5 cm long;
b) an angle of measure 70°.

Angle measure should be restricted to the interval, $0^\circ < \theta \leq 180^\circ$

OBJECTIVES

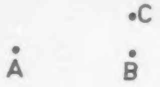
NOTES TO TEACHERS

3. to demonstrate an understanding of the terms collinear, non-collinear, and between;

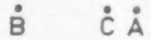
A-C-B can be used to indicate that C is between A and B. Students could demonstrate their understanding by answering questions similar to the following:

In which of the following diagrams is C between A and B?

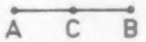
a)



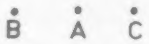
b)



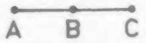
c)



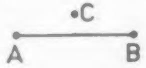
d)



e)

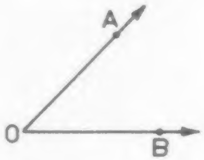


f)



Students must realize that C need not be the midpoint of AB.

4. to demonstrate an understanding of the definitions and terminology associated with angles;



∠AOB has its vertex at O and two sides or arms OA and OB which are rays.

An "angle" could be defined as rotation of a ray about an end point or as a union of two rays with a common end point. Teachers are reminded that technical definitions are of little value unless the symbolism and rigor are maintained.

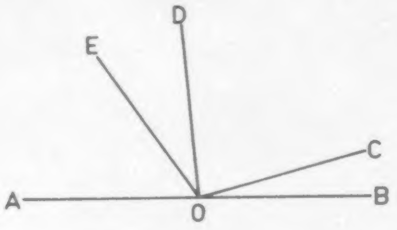
OBJECTIVES

NOTES TO TEACHERS

5. to determine angle measure from a diagram using angle addition and/or subtraction;

Examples:

Use the diagram to answer the questions below.



1) If in A-O-B, $\angle COB = 15°$, $\angle EOC = 110°$ and $\angle EOD = 30°$, what is: a) $\angle DOC$ b) $\angle AOE$ c) $\angle DOB$?

2) If in A-O-B, $\angle EOC = 90°$, $\angle EOD = 3x + 2$ and $\angle DOC = 2x - 12$, find $\angle DOC$ and $\angle EOD$ in degrees.

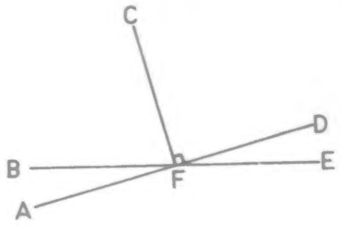
3) Describe $\angle COB$ in terms of $\angle BOE$ and $\angle EOC$.

6. to demonstrate an understanding of and an ability to recognize in diagrams the following types of angles or angle pairs:
   a. acute, obtuse and right,
   b. vertical,
   c. adjacent,
   d. supplementary and complementary,
   e. linear pair;

Vertical angles are referred to as "vertically opposite angles" or "opposite angles" in some textbooks.

Examples:

1) Select an example illustrating angle/angle pairs of each of the types a-e from the following diagram.



OBJECTIVES	NOTES TO TEACHERS
	2) The supplement of an angle is three times the measure of the angle. What is the measure of the angle?
	3) The supplement of an angle is four times the measure of the complement of the same angle. What is the measure of the angle?
7. to demonstrate a understanding of congruence as applied to segments and angles.	Congruence is an optional topic in grade 9 mathematics so it may be a new concept for many students. Note: The line segment AB is congruent to the line segment CD if, and only if, the measure of line segment AB equals the measure of line segment CD. In this guide, $AB \cong CD$ means that line segment AB is congruent to line segment CD. Further, a statement in this guide such as $AB = 3$ means that the measure of line AB is 3 units. If $AB = 3$ and $CD = 3$, then it could be written $AB \cong CD$. Unlike some texts, this guide does not make a careful distinction between the notation for the set of points in a line segment and the number which is the measure of its length. Similar remarks hold for angles.

B. INTRODUCTION TO PROOF (2 periods)

Here is an example of how "proof" could be used in everyday life.

> Sammy is accused of breaking a school window at 8:20 p.m. on Wednesday evening. Sammy and his father were watching the hockey game at home from 7:00 p.m. to 9:30 p.m. on Wednesday evening. Therefore, Sammy could not have broken the window.

Aspects of proof are used in jury trials, computer science, statistical analysis, etc.

> Teachers will have to gauge the amount of rigor to the ability of their students and to the availability of time.

OBJECTIVES

The student should be able:

1. to recognize and to draw conclusions using the properties of equality for real numbers;

2. to recognize and to draw conclusions using the transitive property of congruence for both segments and angles.

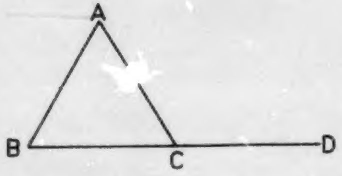
NOTES TO TEACHERS

Example:

John's mass = 60 kg
Joan's mass = 60 kg

∴ John's mass = Joan's mass

Example:



Given: $BC \cong CD$
$CD \cong AC$

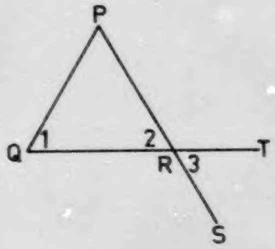
Prove: $BC \cong AC$

Proof:

$BC \cong CD$	given
$CD \cong AC$	given
$BC \cong AC$	Transitive Property

C. USING CERTAIN BASIC GEOMETRIC PROPERTIES TO DEVELOP PROOFS (21 periods)

Properties may be accepted on the basis of their discovery through measurement.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to discover the congruence of vertical angles and to use it in proof questions;	3 periods The congruence of vertical angles may be accepted on the basis of measurement or may be deduced from the properties of supplementary angles. The latter requires different ordering of topics than specified here. Example:  Given: $\angle 1 \cong \angle 3$ Prove: $\angle 1 \cong \angle 2$ Proof: $\angle 2 \cong \angle 3$ (vertical $\angle$'s≅) $\angle 1 \cong \angle 3$ (given) $\angle 1 \cong \angle 2$ (Trans. Prop.) (Assume that $\angle 1 \cong \angle 3$ and that $\angle 3 \cong \angle 1$ are equivalent statements.
2. to demonstrate an understanding of the definition of bisection as relating to segments and angles and to use it in proofs;	3 periods Teachers may wish to introduce the concept of "median" for a triangle in conjunction with the bisection of segments.
3. to demonstrate an understanding of the definition of perpendicularity relating to lines and segments by using this definition in proofs;	3 periods The essential point is that lines must intersect at right angles in order for them to be perpendicular. Teachers may wish to introduce the concept of "altitude" for a triangle in conjunction with perpendicularity.

OBJECTIVES

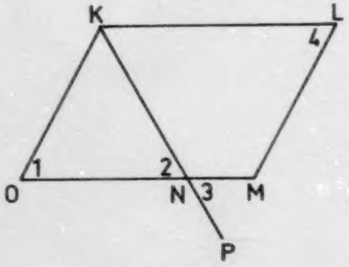
4. to discover the congruence of the base angles of an isosceles triangle and to use it in proofs;

NOTES TO TEACHERS

3 periods

Note: If the order in this guide is followed, the discovery would need to be made using measurement.

Example:



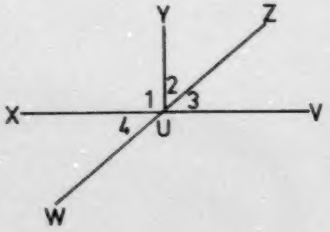
Given: KO ≅ KN
∠1 ≅ ∠4

Prove: ∠3 ≅ ∠4

Proof: KO ≅ KN (given)
∠1 ≅ ∠2 (base ∠'s are ≅ or KO ≅ KN)
∠2 ≅ ∠3 (vertical ∠'s ≅)
∠1 ≅ ∠3 (Trans. Prop.)
∠1 ≅ ∠4 (given)
∠3 ≅ ∠4 (Trans. Prop.)

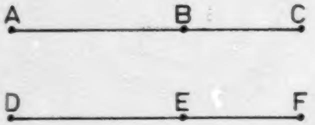
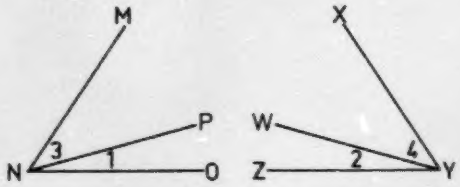
Problems of an algebraic nature could be used at this point, using the properties developed so far.

Example:



Given: UY⊥XV at U
∠YUZ = 40°

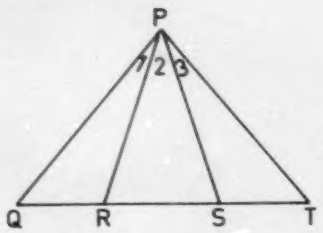
Find: ∠ZUV, ∠4, ∠1, ∠WUV and ∠YUW

OBJECTIVES	NOTES TO TEACHERS
5. to demonstrate an understanding of the Segment Addition and Subtraction Properties by using them in proofs;	3 periods Segment Addition Property:  Let A-B-C and D-E-F as shown. If AB ≡ DE and BC ≡ EF, then AC ≡ DF.
6. to demonstrate an understanding of the interior of an angle and the Angle Addition and Subtraction properties and to use these in writing proofs;	3 periods Example: Angle Addition Property  Let P be in the interior of ∠MNO, and W be in the interior of ∠XYZ. If ∠1 ≡ ∠2 and ∠3 ≡ ∠4, then ∠MNO ≡ ∠XYZ.

OBJECTIVES

NOTES TO TEACHERS

Example:



Given: $\angle 1 \cong \angle 3$

Prove: $\angle QPS \cong \angle RPT$

Proof: $\angle 1 \cong \angle 3$ (given)
$\angle 2 \cong \angle 2$ (common)
$\angle QPS \cong \angle RPT$ (Angle Addition Property)

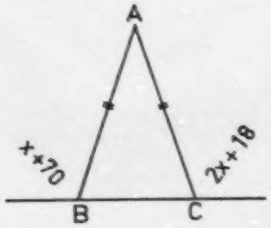
7. to use the concepts of linear pair, supplementary angles and complementary angles in proofs.

3 periods

"Supplements of congruent angles are congruent" and "Complements of congruent angles are congruent" are theorems which students should prove.

Algebraic exercises that involve solving equations should be used here.

Example:



Given: Diagram with segments and angles as marked.

Find: x, $\angle ABC$

D. USING THE CONGRUENCE OF TRIANGLES TO DEVELOP PROOFS (15 periods)

OBJECTIVES

The student should be able:

1. to demonstrate an understanding of a congruence correspondence between two triangles;

NOTES TO TEACHERS

2 periods

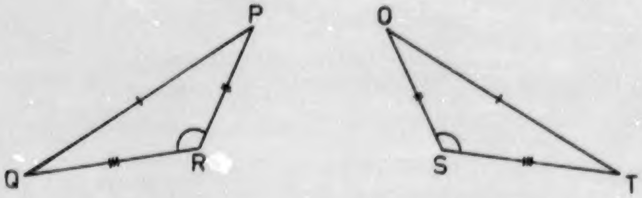
If △ABC ≅ △DEF, then A corresponds to D, B corresponds to E, and C corresponds to F, so that ∠ABC ≅ ∠DEF, ∠ACB ≅ ∠DFE, ∠BAC ≅ ∠EDF, AB ≅ DE, BC ≅ EF and AC ≅ DF.

The emphasis in this first objective is for students to pick out the congruent angles and sides, identifying the vertices in the correct order.

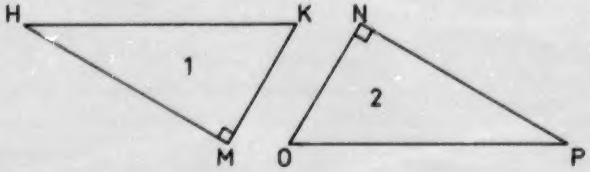
Examples:

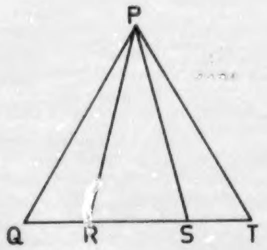
1) If △ABC ≅ △DEF, then name the congruent sides and angles.

2) Name the congruent sides and angles in the accompanying diagram.

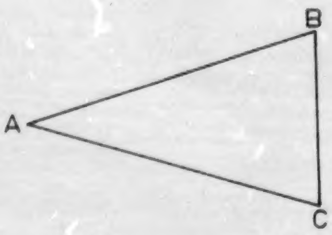


3) If triangle 1 and triangle 2 are congruent, name the congruent triangles, sides
angles.



OBJECTIVES	NOTES TO TEACHERS
2. to postulate the SAS congruence property, to use it to prove the congruence of triangles, and to name the remaining corresponding congruent side and angles of the triangles;	4 periods Following construction and measurement, teachers are expected to postulate SAS, SSS and ASA congruences. Example: Draw a segment 6 cm long and call it KP. With your protractor, draw an angle of 50° at K and name point M 4 cm along the new side of the angle. Join P and M. Measure ∠P, ∠M and PM. Repeat this performance. Compare the measures of corresponding angles and sides.
3. to postulate the SSS congruence property, to prove triangles congruent, and to identify corresponding congruent angles;	3 periods Example:  Given: PR ≅ PS PQ ≅ PT QR ≅ TS Prove: ∠QPS ≅ ∠TPR Proof: PR ≅ PS (given) PQ ≅ PT (given) QR ≅ TS (given) RS ≅ RS (common) QS ≅ TR (Seg. Add.) △PQS ≅ △PTR (SSS) ∠QPS ≅ ∠TPR (corr. parts)
4. to postulate the ASA congruence property, to use it to prove the congruence of triangles, and to name the remaining corresponding congruent angle and sides of the triangles;	3 periods

OBJECTIVES	NOTES TO TEACHERS
5. to write the converse of a given statement having the form "if p, then q";	I period Students should be aware that if a given statement is true, then its converse is not necessarily true. Example: Write the converse of each of the following statements and indicate whether this converse is true or false: 1) If two triangles are congruent, then the triangles have equal area. Converse: If two triangles have equal area, then the triangles are congruent. (This converse is not true since we can find a counter example.) 2) If a triangle is isosceles, then two of its angles are congruent. Converse: If two angles of a triangle are congruent, then it is isosceles. (True.)
6. to prove the converse of the following statement and to use it in proofs: "If a triangle is isosceles, then two of its angles are congruent."	2 periods



Given: $\angle ABC \cong \angle ACB$

Prove: ΔABC is isosceles

Proof: Consider the correspondence ABC and ACB

$\angle ABC \cong \angle ACB$	(given)
$BC \cong CB$	(definition of congruence)
$\angle ACB \cong \angle ABC$	(given)
$\Delta ABC \cong \Delta ACB$	(ASA)
$AB \cong AC$	(corresponding sides of $\cong$ Δ's)
ΔABC is isosceles	(definition)

Note: This proof is somewhat subtle and may not be understood by many students. Less abstract proofs are possible after more properties are developed (e.g. triangle sum, A.A.S.).

Work on parallel lines is part of Mathematics 200 and is not intended as part of this course.

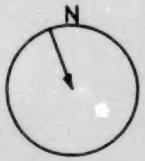
GEOMETRY OPTION

Do one of either Op.I Vectors or Op.II Orienteering

Op.I. VECTORS (½ module)

A. INTRODUCTION

This unit deals with two-dimensional vectors. Since vector subtraction has not been considered, problems where the resulting or final situations are defined and one of the constituent vectors has to be found will have to be avoided.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to distinguish between vector and scalar quantities;	Vector quantities possess both size and direction. Velocity, acceleration and force are examples of vectors. Vectors are identified by the use of arrows. Scalars do not possess direction. Quantities of things like time, money, etc. are scalars.
2. to interpret and to use compass directions;	Common systems in use include: 1) One that uses only North as the reference direction. Directions are then given from North (assumed) measured clockwise. Examples: a) Wind at 340° implies that the wind is coming from 340° as in the diagram.  b) Wind at 015° implies wind coming from 15°. (Note that three digits are always used in this system.)

OBJECTIVES

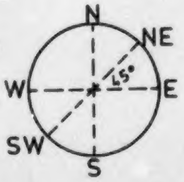
3. to name and to represent on diagrams vectors using:

   a. magnitude and direction,

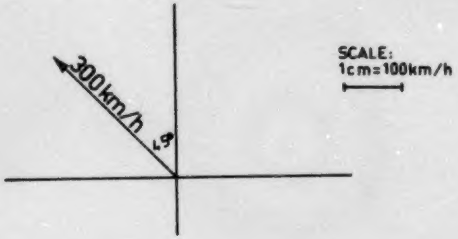
   b. geometric and algebraic representations,

NOTES TO TEACHERS

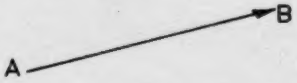
2) One that uses N, NE, E, etc., as exact directions. Between these, degree measures are used to identify directions, for example, 15° E of N.



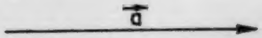
For example, if a plane is travelling at 300 km/h to the NW, a suitable scale is set up and identified and the direction noted in the diagram.



Examples:



vector $\overrightarrow{AB}$



vector $\vec{a}$

OBJECTIVES

c. ordered pairs.

B. EQUAL VECTORS

The student should be able to define and to identify equal vectors.

C. VECTOR ADDITION

The student should be able to add two vectors.

NOTES TO TEACHERS

In this case, the starting point and the terminal point of the vector is identified.

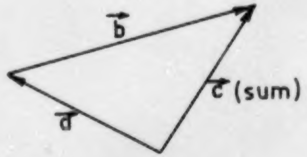
Examples:

1) the vector from (3,5) to (7,2)
2) the vector from (0,0) to (-3,-2)

Note: Vectors written as ordered pairs will not be considered further in this course.

Two vectors are equal if and only if they have the same magnitude and direction. They need not have the same starting and ending point.

Two vectors can be added by placing the tail of one vector at the head of another. (Any vector that is parallel to the first and has the same magnitude is considered to be the same as the first.) When the third side of the resulting triangle is drawn, it represents the sum vector. Direction is as shown.



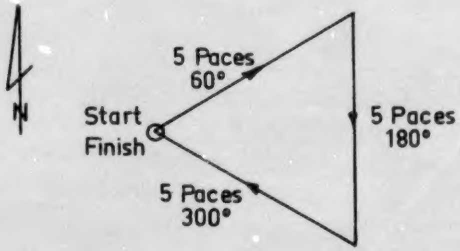
$\vec{a} + \vec{b} = \vec{c}$ (sum)

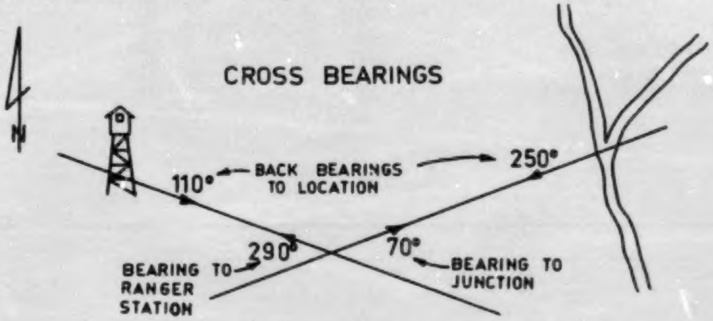
OBJECTIVES	NOTES TO TEACHERS
D. VECTOR MULTIPLICATION The student should be able:	
1. to multiply a vector by a number;	Consider topics both arithmetically and graphically. The magnitude of the vector is multiplied, but the direction is unchanged; i.e., (15 km/h East) x 5 = 75 km/h East.
2. to multiply a vector by a scalar (with units).	The magnitude of the vector and the nature of the vector are changed, but again, the direction remains the same. (15 km/h East) (3 h) = 45 km East; i.e., $(\overrightarrow{\text{velocity}})(\text{time}) = \overrightarrow{\text{displacement}}$.
E. VECTOR PROBLEMS The student should be able to use vector addition and scale drawings to solve relevant vector problems.	Examples could involve aircraft and wind problems, boat and current problems, swimmer and current problems, or force problems.

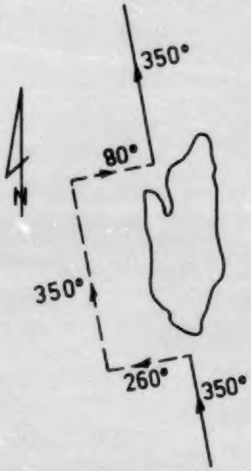
GEOMETRY OPTION

Op.II. ORIENTEERING (½ module)

In the past, men and women have used the sun, moon, stars, prevailing winds, currents, waves and the magnetic field to find directions. Knowledge of this skill could prove useful for students.

OBJECTIVES	NOTES TO TEACHERS
A. ORIENTEERING ON THE SCHOOL GROUNDS The student should be able:	
1. to demonstrate an understanding of the magnetic compass;	The basic N, E, W and S directions should be known, as well as the 360 division.
2. to find the bearing of a visible object;	Objectives (2) and (3) could be taught on the school grounds. Standing at home plate on the baseball diamond, students could determine the bearing of an object, say the left corner of the green house. Standing at the same spot, they could identify an object at a certain bearing, say of 80°.
3. to find the direction of a given bearing;	
4. to measure distances using the number of paces (double step) required for 100 metres;	An example of an activity which involves both objectives (3) and (4) is the attempt to walk an equilateral triangle, marking each point by some means and ending up at the start. This can be done by walking 5 (or 10) paces at 60°, then 5 (or 10) paces at 180°, and then 5 (or 10) paces at 300°.  N 5 Paces 60° Start Finish 5 Paces 180° 5 Paces 300°

OBJECTIVES	NOTES TO TEACHERS
5. to find a station, given a starting point, a bearing, and a distance;	Several starting points could be set up on one side of a field. The distances and bearings to several other stations, on the other side of the field, should be pre-measured before students begin this activity.
6. to follow an orientation trail from one station to the next, given the bearing and distance between each station.	This exercise could be laid out either in the school grounds or in a local park.
B. ORIENTEERING WITH A MAP Students should be able:	
1. to read a contour map and to be able to interpret all the aids and obstacles necessary for following an orienteering line;	Contours can indicate slopes and their relative steepness. Rivers, lakes and marshes can also be recognized. Roads, paths and buildings are definite aids in trying to follow or read a map.
2. to demonstrate an understanding of magnetic declination, back bearings and cross bearings;	Magnetic declination changes from one area of the world to another. Back bearings can be used to check the line being followed and, together with cross bearings, can be used to find your position on a map, given visible landmarks that are noted on the map.  CROSS BEARINGS N 110° BACK BEARINGS TO LOCATION 250° 290° 70° BEARING TO RANGER STATION BEARING TO JUNCTION Note: The cross bearings originate from the land marks and are drawn by using the back bearings of each for a given location.

OBJECTIVES	NOTES TO TEACHERS
3. to demonstrate the ability to detour around an object or obstacle and to get back on the same bearing as before the detour;	A relatively easy way to make a detour is to walk a rectangular path around the obstacle, subtracting 90° from the bearing when turning left and adding 90° to the bearing when turning right. 
4. to follow an orienteering route marked out on a map, with a separate sheet identifying the bearing and the distance between each station;	This exercise should be carried out in a park or rural area for which a contour map is available. Such an area will ensure that the students will be out of each other's sight, thus making the exercise a more valuable individual experience. Such courses should be set up well in advance and checked by another person.
5. to follow an orienteering route with only the stations marked on a map.	This exercise should also be planned with care. A possible alternative could be to have students take part in a regular orienteering meet organized by the Manitoba Orienteering Association.

CONSUMER MATHEMATICS

Do one of either Op.III Money Management or Op.IV Consumer Statistics. While some teachers may prefer to deal primarily with the mathematics of the situation(s) and others with consumerism in relation to life skills, it is recommended that a balanced approach be used, and even that emphasis be placed on individual consumer aspects of the objectives under consideration.

Teachers may wish to teach the consumer option throughout the school term rather than as a separate module.

Op.III. MONEY MANAGEMENT (1 module)

OBJECTIVES	NOTES TO TEACHERS
A. INCOME FROM EMPLOYMENT	
The student should be able:	
1. to identify and to solve problems associated with types of income;	Types of income include hourly, weekly, or monthly wages; overtime; piecework; commission and tips. Example: A salesperson is offered a job in which he/she would receive a salary of $100 a week plus 9% of sales. How much merchandise must he/she sell on the average per week to earn $20 000 per year?
2. to calculate an individual's net periodic pay given the appropriate rates/amounts of deductions;	Calculations of deductions such as for income tax, unemployment insurance, Canada Pension Plan, etc. could be dealt with. Schools with access to computers may consider existing programs designed to calculate pay cheque amounts.
3. to complete a basic income tax return.	Tax forms and other relevant materials are available from Revenue Canada and local post offices. Because the income tax return and guide are revised annually, it is suggested that teachers obtain the publication Teaching Taxes by writing to: Teaching Taxes Information Services Branch Revenue Canada, Taxation 875 Heron Road Ottawa, Ontario K1A 0L8

OBJECTIVES | **NOTES TO TEACHERS**

B. BANKING AND INTEREST

The student should be able:

1. to perform calculations involving simple interest;

The formula I = Prt, where I = interest, P = principal, r = rate of interest per annum and t = time in years, can be illustrated using problems.

Examples:

1) Find the simple interest on $1000 for 6 months when the interest rate is 15% per year. What is the accumulated amount at the end of 6 months?

2) Find the simple interest rate per year at which $1000 will amount to $1120 at the end of 9 months.

3) How long will it take for $200 to amount to $209 at a simple interest rate of 18% per year?

2. to perform calculations involving compound interest.

The formula $S = P(1+i)^n$, where S = accumulated amount, P = principal, i = rate of interest per interest period and n = number of interest periods, can be illustrated using problems. Examples could include:

1) $1000 is invested in a bank account paying 18% compounded quarterly. How much money will be in the account at the end of 2 years?

2) How much money must be deposited in a bank account paying 18% compounded monthly to accumulate to $1000 at the end of 4 years? (This illustrates the meaning of present value.)

Note: Calculations involved in these problems should be done with a hand calculator. Students who do not have access to one should use tables for compound interest.

If time permits, teachers may also want to consider annuities.

Following are some suggestions of relevant topics that could be dealt with as class projects:

a) A survey of the various types of savings accounts, chequing accounts, and combination accounts available at banks, savings and loan associations, and credit unions, with a discussion of the advantages and disadvantages of each.

OBJECTIVES	NOTES TO TEACHERS
	b) Savings accounts Advantages of a savings account Methods used in computing interest Calculation of periodic interest for an account Completion of deposit and withdrawal slips Entry and verification of passbook transactions
	c) Chequing accounts Advantages of using cheques for payments Chequing charges How to write a cheque Keeping records of all transactions affecting an account Reconciling a personal statement with a monthly bank statement Finding mistakes and seeking their correction
C. CREDIT BUYING The student should be able to perform calculations involving credit buying.	Example: A man buys a used car priced at $2500 by making a $500 down payment and agreeing to pay 30 monthly payments of $80 each. Estimate the effective interest rate. For one method of estimating an effective interest rate, refer to : E.G. Carll, J.C. Egsgard, and C. Psica, Mathematics for a Modern World, Book 2, second ed. (Toronto: Gage, 1975), p. 376.

Op.IV. CONSUMER STATISTICS (I module)

OBJECTIVES

The student should be able:

1. to organize a given set of data derived from a discrete variable into an ungrouped frequency table;

NOTES TO TEACHERS

Example:

Thirty fishermen were checked one day by the Department of Fisheries. A tally of the number of fish in each one's possession follows:

7	5	9	3	5	4	2	3	1	7
3	2	4	1	8	6	5	4	3	4
10	9	2	4	2	1	4	7	12	3

Students are expected to make an ungrouped table much like the following:

Table I

Score	Tally Marks	Frequency
1	|||	3
2	||||	4
3	~~||||~~	5
4	~~||||~~ |	6
5	|||	3
6	|	1
7	|||	3
8	|	1
9	||	2
10	|	1
11		0
12	|	1
	TOTAL	30

OBJECTIVES

2. to classify given ungrouped discrete data and to make a grouped frequency table;

3. to name the exact limits of a given class interval, and to group a given set of data derived from a continuous variable into a table;

NOTES TO TEACHERS

For tables with integral data, as in the previous example, grouping can be done simply as below:

Table 2

Score	Tally Marks	Frequency
1-3	~~IIII~~ ~~IIII~~ II	12
4-6	~~IIII~~ ~~IIII~~	10
7-9	~~IIII~~ I	6
10-12	II	2
	TOTAL	30

For data involving rational values, students must choose the end-points of their intervals carefully. Even if the variable under consideration is continuous and not discrete, observations must be recorded as discrete values. Each recorded value is implied to be between two limits and these limits are usually taken to be one-half a unit above and one-half a unit below the reported value.

For instance, if a race is being timed to the nearest hundredth of a second, a time recorded as 10.33 seconds would be understood to be between 10.325 s and 10.335 s. In fact, by convention, if "t" is the time recorded, $10.325 \leq t < 10.335$.

If the data in Table 2 is taken to be continuous, the following changes could be made:

Table 3

Score	Exact Limits	Mid-point of Interval	Frequency
0-3	0.5 - 3.5*	2.0	12
4-6	3.5 - 6.5	5.0	10
7-9	6.5 - 9.5	8.0	6
10-12	9.5 - 12.5	11.0	2
		TOTAL	30

*Note the extension of the first class.

OBJECTIVES

4. to construct a histogram and a frequency polygon to illustrate a given set of data;

NOTES TO TEACHERS

There are numerous conventions regarding the selection of class intervals. Some of these include:

1) Select the size of interval, so that between 10 and 20 intervals will cover all observations.

2) Try to select intervals with ranges of 1, 2, 3, 5, 10 or 20 points.

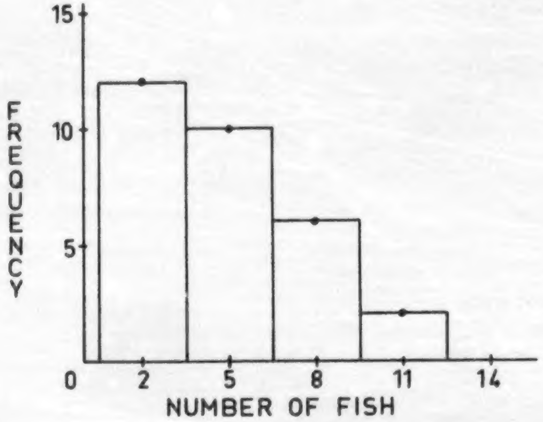
3) Start the class interval at a value which is a multiple of the size of the interval.

Example:

The data from the previously developed example could be presented in either of the following graphs:

Graph I

Histogram Showing the Number of Fish in the Possession of Fishermen.

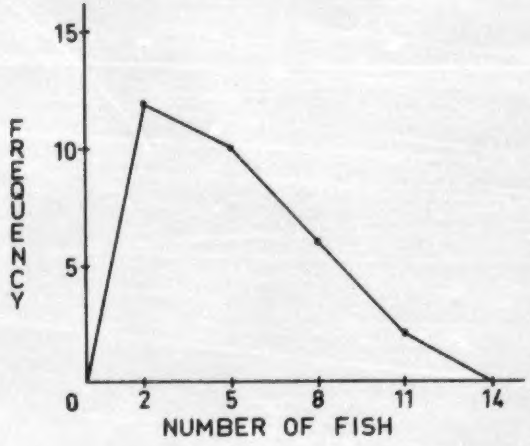


OBJECTIVES

NOTES TO TEACHERS

Graph 2

Frequency Polygon Showing the Number of Fish in the Possession of Fishermen.



5. to identify sources of bias related to a given graph's structure;

Teachers can find good examples for this and other topics in books such as How to Lie with Statistics by Darrell Huff (N.Y.: W.W. Norton, 1954) and Mathematics: A Human Endeavor by Harold Jacobs (San Francisco: W.H. Freeman, 1970).

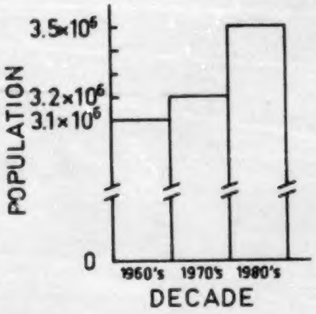
Also refer to Op.VIII, Graphing, in the Mathematics 101 outline.

Truncation of graphs and changes in scale can make the information presented appear significant in ways not necessarily warranted by fact.

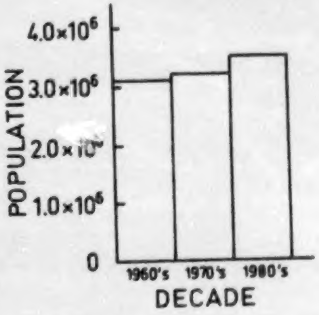
Examples:

1. Both graphs indicate the rise in population of a country over a period of time, yet the rise may seem significant in Graph 3 and relatively insignificant in Graph 4.

Graph 3

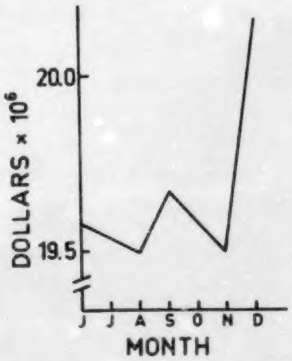


Graph 4

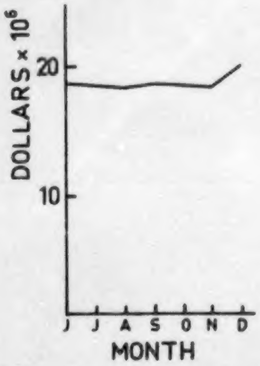


2. Different impressions result when different vertical scales are used to graph government expenditures over a period of time.

Graph 5



Graph 6



OBJECTIVES

6. to calculate the value of the measures of central tendency (averages) including:

   a. mean,
   b. median,
   c. mode;

NOTES TO TEACHERS

Example:

You and a friend play 9 holes of golf and get the following scores:

Table 4

Hole	1	2	3	4	5	6	7	8	9	Total
You	4	6	3	4	15	3	4	3	3	45
Friend	6	7	5	5	5	4	5	3	4	44

Who has the best "average" score?

1) Mean: Yours $\frac{45}{9} = 5$ Friend's $\frac{44}{9} = 4.9$

This "average" indicates your friend is the better golfer.

2) Median: First arrange the scores in order of size.

Table 5

Yours	Friends	
3	3	
3	4	
3	4	
3	5	
4	5	Median score (Middle score)
4	5	
4	5	
6	6	
15	7	

Since your middle score is lower than your friend's, this "average" indicates you are the better golfer.

Note: If an even number of scores is tallied, the median score is half-way between the middle two.

OBJECTIVES

7. to choose the "average" which could be used to greatest advantage for a given purpose;

8. to determine the range of a given set of data;

9. to devise a sampling technique which will avoid obvious sources of bias;

10. to pose an experimental statistical problem, to design the data-gathering methods, to gather the data, and to present it appropriately using methods developed in this module.

NOTES TO TEACHERS

3) Mode: From the preceding table, it is clear that you shot a 3 most often. This is your mode. Your friend's is 5. Using this "average," you again appear to be the better golfer.

Examples:

1) Which of the "averages" identified above represents the scores of you and your friend in the fairest way?

2) A small clothing industry has 5 workers who are paid $8000 each, one maintenance worker and one supervisor getting $12 000 each, one designer and one salesman getting $20 000 each, and an owner who pays himself $50 000. (All salaries are stated on a per annum basis.)

   Which way would you represent the "average" pay of this industry if you were:

   a) the owner?
   b) the union shop steward?

If time permits, a discussion of the shortcomings of measures of central tendency (averages) could lead to a discussion of measures of dispersion (variance and standard deviation) and the normal curve.

For example, ask when or where students conducting a poll to predict the outcome of the student council elections would pose their questions--only during Math class everyday for a week (when other students consistently engage in the same activities), or only in a certain location (when segments of the student body may avoid that location)?

The teacher may wish to spend some time after the experiments have been done discussing the difficulties involved in attempting to draw conclusions or to make predictions from collected data.